Diffraction as scattering under the Born approximation

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INTRODUCTION

Diffraction of light through an aperture is a well-known and significant phenomena in optical science. Diffraction formalism to explain the diffraction gave rise to several solutions, the most well accepted solutions are the Fresnel-Kirchhoff and Rayleigh-Sommerfeld formulas [1-2]. Each expression differs from the other based on the solution of wave equation under the assumption of different Green's functions and different sets of boundary conditions. While these formulas agree well in the far-field, they suffer from discrepancies and non-agreement in the near-fields due to the different obliquity factors involved.

Here, we summarize the results of our work in Ref. [4] where we report a mathematical treatment of diffraction through thin apertures like grating as a scattering problem Under the first order Born approximation to 2D objects, we formulate the diffraction solution without any angular approximations. Our result indicates that diffraction formulas with unity obliquity factor can be achieved to explain diffraction through scattering under first-order Born approximation.

In this study, we aim to:

- Combine the explanation of diffraction through phase objects and scattering under a unified formalism.
- Solve the ambiguity caused by obliquity factors in different diffraction formulas by approaching the problem of diffraction through a thin object under Born approximation through scattering theory.

Diffraction through phase objects can be modelled as a scattering phenomena.

THEORY

Approximating the refractive index distribution of the object as per figure 1 as [4]

$$n^2(\mathbf{r})-1=\left[n^2(\mathbf{r})-1\right]\Pi\left(\frac{z}{\Delta z}\right) \qquad [1]$$

where, Π function represents a thin slice of width Δz which for an infinitesimally thin phase object can be simplified to [4]

$$n^2(\mathbf{r})-1\simeq\left[n^2(\mathbf{r}_\perp)-1\right]\Delta z\,\delta z \qquad [2]$$

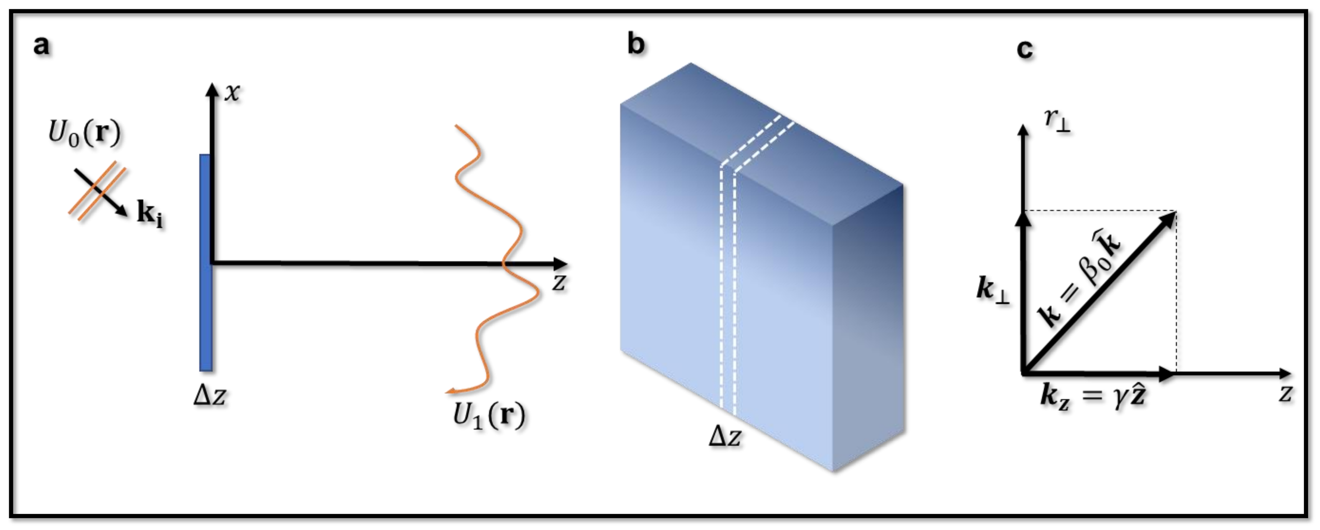

Fig. 1 Scattering geometry. Ref.4

Object transfer function can then be modelled as [4]

$$t(\mathbf{r}_\perp)=\beta_0\left[n^2(\mathbf{r}_\perp)-1\right]\Delta z \qquad [3]$$

Solving the Helmholtz equation with the object represented by the proposed transfer function [3,4]

$$(\nabla^2+\beta_0^2)U(\mathbf{r};\omega)=-\beta_0\delta(z)t(\mathbf{r}_\perp)U(\mathbf{r};\omega) \qquad [4]$$

On applying first-order Born approximation, the solution in angular spectrum domain is [4]

$$U_1(\mathbf{k}_\perp,z;\omega)=i\beta_0\left[t(\mathbf{k}_\perp)\circledcirc_{\mathbf{k}_\perp}U_0(\mathbf{k}_\perp,0;\omega)\right]\frac{e^{i\gamma(\mathbf{k}_\perp)z}}{2\gamma(\mathbf{k}_\perp)}. \qquad [5]$$

Expressing the solution in spatial domain gives [4]

$$U_1(\mathbf{r}_\perp,z;\omega)=\beta_0A(\omega)\left[U_0(\mathbf{r}_\perp,0;\omega)t(\mathbf{r}_\perp)\circledcirc_{\mathbf{r}_\perp}\frac{e^{i\beta_0r}}{r}\right] \qquad [6]$$

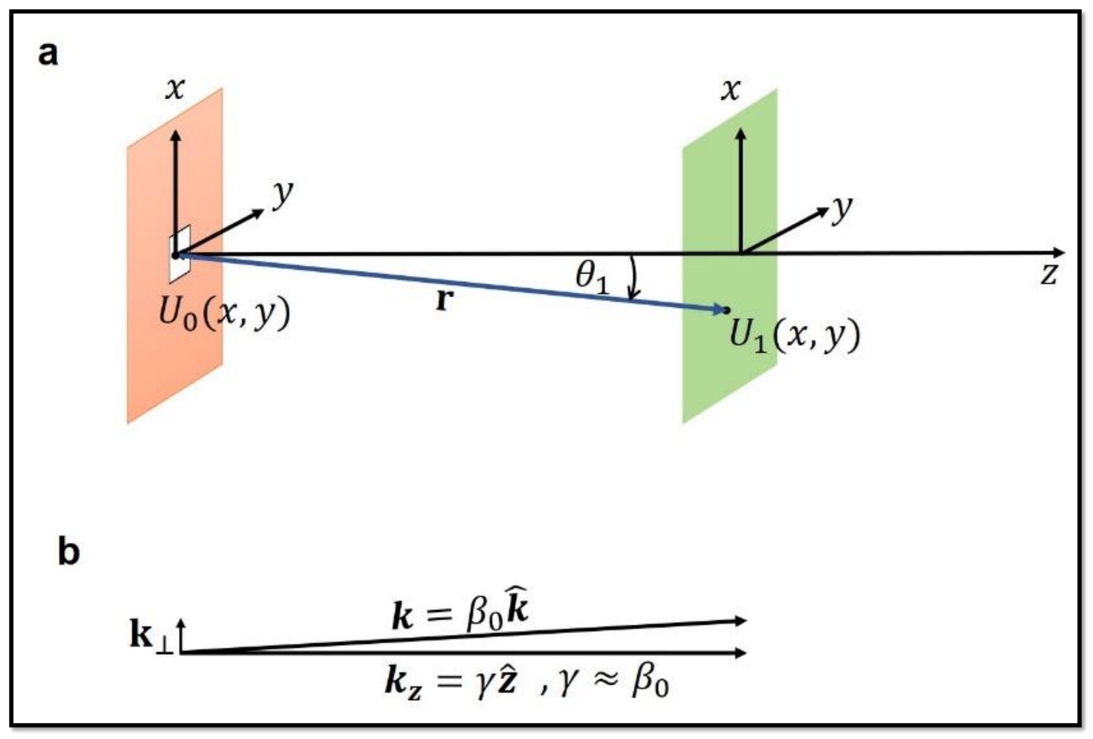

Fig. 2 Geometry for angular approximations. Ref. 4

Next, we assume plane wave incidence and apply various approximations

First, we apply Fresnel approximation for small angle as shown in Fig 2, under which the solution reduces to [4]

$$U_1(\mathbf{r}_\perp,z;\omega)=\frac{i}{2}A(\omega)e^{i\beta_0z}\left[e^{i\beta_0\left(\frac{r_\perp^2}{2z}\right)}\circledcirc_{\mathbf{r}_\perp}t(\mathbf{r}_\perp)\right] \qquad [7]$$

Further applying Fraunhofer approximation gives the following solution

$$U_1(x,y,z;\omega)=\frac{i}{2}A(\omega)e^{i\beta_0z}e^{i\beta_0\left(\frac{x^2+y^2}{2z}\right)}t(k_x,k_y) \qquad [8]$$

$$k_x=\beta_0x/z,\quad k_y=\beta_0y/z$$

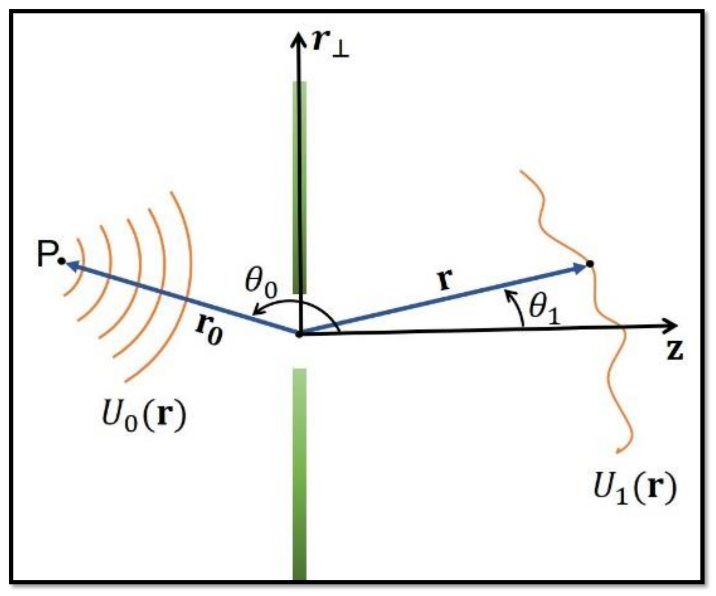

Fig. 3 Diffraction of a spherical wavefront by an aperture. Ref. 4

For the case of a single point source emitting spherical waveform as shown in fig 3 and following the theoretical framework discussed above, the diffracted field can be expressed as

$$U_1(\mathbf{r})=\iint_a\frac{e^{i\beta_0|(\mathbf{r}'_\perp,0)-\mathbf{r}_0|}}{|(\mathbf{r}'_\perp,0)-\mathbf{r}_0|}\frac{e^{i\beta_0|\mathbf{r}_\perp-\mathbf{r}'_\perp,z|}}{|\mathbf{r}_\perp-\mathbf{r}'_\perp,z|}d^2\mathbf{r}'_\perp=\frac{A}{i\lambda}\iint_a\frac{e^{i\beta(r_{21}+r_{01})}}{r_{21}r_{01}}ds \qquad [9]$$

This result for the diffraction of a spherical wavefront by an aperture matches the well-known diffraction equations with *unity obliquity factor*.

CONCLUSIONS

1. **This work lays the theoretical foundation to combine scattering and diffraction formalisms for apertures that can be considered thin phase objects**
2. **The results match with the conventional diffraction formulas with the exception of an obliquity factor which turns out to be unity through scattering solution.**

REFERENCES

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