Ref ETOP098

Slow Photons in Vacuum as Elementary Particles

Chander Mohan Singal

Department of Physics, Indian Institute of Technology-Delhi, Hauz Khas,
New Delhi-110016, INDIA .   E-Mail: drcmsingal@yahoo.com

Abstract
Maxwell equations for Electro-magnetic(EM) vector fields **E** and **B** are solved in vacuum, free from charges and currents, and EM wave packets propagating in z direction are formed using cylindrical coordinates (s, φ, z) with an average propagation vector k_0 and a radial parameter γ used for defining Bessel function J_n (γ s) of n'th order . It is shown that these wave packets have a group velocity $v_g = ck_0 / [k_0^2 + \gamma^2]^{0.5}$ smaller than the standard velocity c of light in vacuum, and they spread in z direction with progression of time, like particles having non-zero rest mass $m_0 = \hbar \gamma / c$. It is shown that these slow photons can describe motion of elementary particles like electrons and protons with regard to their velocity and linear momentum . Quantization of energy U of these EM wave packets is done using the condition $U = \hbar c [k_0^2 + \gamma^2 + \sigma^2]^{0.5}$ and then the angular momentum determined for them, (here σ is the standard deviation in the propagation vector k). After quantization, the z components of the linear momentum and angular momentum of the wave packets are found to be $\hbar k_0$ and n $\hbar$, respectively. It is shown that for γ much smaller than k_0 these wave packets can appear like light photons , and for γ much larger than k_0 these wave packets can appear like electrons and protons, with regard to their mechanics .

Key Words
Slow Photon, Electron, Rest Mass, Energy, Velocity, Linear   Momentum , Angular Momentum.

Summary
Mathematical Formulation:
Electro-magnetic (EM) fields **E** and **B** are well known[1] to follow Maxwell equations giving divergence and curl of the vectors **E** and **B** . Plane wave solutions are normally obtained by solving these equations in Cartesian coordinates, and velocity of these plane waves in vacuum is known to be c = $2.99792458 \times 10^{10}$ centimeter/second . However, the equations for fields **E** and **B** can be solved in cylindrical coordinates (s, φ, z) as well, using functions of the type $J_n(\gamma s)$ exp(ikz + inφ – iωt) where n is the order of the Bessel function J_n with radial coordinate s and Bessel parameter γ, k is the wave propagation vector in z direction, and ω is the angular frequency given by
$(\omega/c)^2 = k^2 + \gamma^2$. In the present study wave packets have been formed in vacuum (free from charges and currents) using these solutions, and the time dependent motion of these wave packets is studied numerically with the help of a high speed computer. In the following analysis the Gaussian system of units is utilized for the EM fields. Maxwell equations[1] for **E** and **B** fields in vacuum, free of charges and currents, are :
$\nabla \cdot \mathbf{E} = 0$; $\nabla \cdot \mathbf{B} = 0$; $\nabla \times \mathbf{E} = - (\partial \mathbf{B} / \partial t) / c$; $\nabla \times \mathbf{B} = (\partial \mathbf{E} / \partial t) / c$.
(1)
Each Cartesian component ψ of **E** and **B** satisfies the wave equation :
$$\nabla^2 \psi = (\partial^2 \psi / \partial t^2) / c^2 .$$
(2)
For the cylindrical coordinates, the z component of the **E** and **B** fields are the same as those for the Cartesian coordinates and thus for E_Z or B_Z the solution is taken from the above

wave equation. The remaining components E_S, E_ϕ, B_S and B_ϕ are then evaluated directly from Maxwell equations. For transverse magnetic (TM) mode B_Z is zero, and for transverse electric (TE) mode E_Z is zero. For TE mode, one set of real electric and magnetic field components in cylindrical coordinates is:

$(\textbf{TE})_1$:

$B_Z = A\ (2\pi R/\gamma)^{-0.5}\quad J_n\quad \int\ \cos(\theta)\ [g(k)]^{0.5}\ dk$;
$B_S = -A\ (2\pi R\gamma)^{-0.5}\quad (\partial J_n/\partial s\ )\quad \int\ [\ k\ /\ \gamma]\ \sin(\theta)\ [g(k)]^{0.5}\ dk$;
$B_\phi = -A\ (2\pi R\gamma)^{-0.5}\quad n\ (J_n/s\ )\quad \int\ [\ k\ /\ \gamma\ ]\ \cos(\theta)\ [g(k)]^{0.5}\ dk$;
$E_S = -A\ (2\pi R\gamma)^{-0.5}\quad n\ (J_n/s\ )\quad \int\ [\ \omega\ /(c\ \gamma\ )]\ \cos(\theta)\ [g(k)]^{0.5}\ dk$;
$E_\phi = \quad A\quad (2\pi R\gamma)^{-0.5}\quad (\partial J_n/\partial s)\quad\quad \int\ [\ \omega\ /\ (c\ \gamma\ )]\quad \sin(\theta)\quad [g(k)]^{0.5}\quad dk$.
(3)

where $\theta = (\ kz + n\ \phi - \omega t\ )$, A is an amplitude factor, R is the large radius of the cylinder within which the Bessel function is normalized . For forming the wave packet, square root of the Gaussian function g(k) has been multiplied with each of the above field components and integration carried out over the range of k values from $-\infty$ to $+\infty$, with g(k) as :

$g(k)\quad =\quad [\quad 1\quad /\quad (\sigma\quad (2\pi)^{0.5}\quad)\quad].\quad \exp[\quad -\quad (k-k_0)^2\quad /\quad (2\quad \sigma^2\quad)\quad]$.
(4)
Another complementary set of EM fields denoted by $(\textbf{TE})_2$ is obtainable from the field set (3) by interchange of the $\cos(\theta)$ and $\sin(\theta)$ factors. In the present analysis, to begin with for simplicity, the order n = 0 is considered for numerical evaluations. This then is taken as a representation of a photon described through its EM field set. The energy density of the EM field for the TE mode at any coordinate point (s , ϕ , z) at any time t is then u , given by :

$u\quad =\quad (\quad B_Z^2\quad +\quad B_S^2\quad +\quad B_\phi^2\quad +\quad E_S^2\quad +\quad E_\phi^2\quad)\quad /\quad (8\ \pi\ )$.
(5)
Further integration of this energy density u is carried out over the complete range of s from 0 to R and for ϕ from 0 to 2π , so that study of variation of energy density along the z axis can be carried out. The energy densities obtained for the field sets $(\textbf{TE})_1$ and $(\textbf{TE})_2$, are given symbols U_{Z1} and U_{Z2} , and average of these energy densities is then written as U_Z . For n = 1 and higher orders the values of U_{Z1} and U_{Z2} are found to be equal and have the same analytical form as for the U_Z obtained for the n = 0 order. Thus , in general

$U_Z\quad =\quad A^2\quad .\quad \{\ f_1\ (\ z\ ,\ t\ )\quad +\quad f_2\ (\ z\ ,\ t\ )\quad \}\quad /\quad (16\pi^2\ )$; where
(6)
$f_1\ (z,\ t) = \{\ \int \cos(\theta)\ [g(k)]^{0.5}\ dk\ \}^2 + \{\ \int (k/\gamma\ )\ \sin(\theta\ )\ [\ g(k)]^{0.5}\ dk\ \}^2 + \{\ \int [\omega/(c\gamma)]\ \sin(\theta)\ [g(k)]^{0.5}dk$ $\}^2$; $f_2\ (z,\ t) = \{\ \int \sin(\theta)\ [g(k)]^{0.5}\ dk\ \}^2 + \{\ \int (k/\gamma\ )\cos(\theta\ )\ [\ g(k)]^{0.5}\ dk\ \}^2 + \{\ \int [\omega/(c\gamma)]\ \cos(\theta)\ [g(k)]^{0.5}dk\ \}^2$;

with $\theta = (kz - \omega t)$. Further, integration of the U_Z over z from $-\infty$ to $+\infty$ gives the energy U of the photon, upon also completing the k integration as:

$U=A^2[k_0^2+\gamma^2+\sigma^2]\qquad\qquad /\qquad\qquad [4\pi\gamma^2\qquad]$.
(7)

Computational Results and Application to Laser Beams and Massive Particles:
Numerical evaluation of U_Z for the TE mode is done from equation (6), with value of c taken as 1 and amplitude A also taken as 1, for ease of calculation. The photon parameters are taken as:

$k_0 = \pi$, $\gamma = \pi$, $\sigma = 0.1\pi$. Figure 1 shows the variation of U_Z with z at t = 0. It is seen from the figure that the maxima of energy density of the EM field is located near z = 0. Figure 2 shows the variation of U_Z with z at times t = 0, 5, 10, 15, and 20 seconds (as series 1, 2, 3, 4 and 5 respectively). The location of the peak in energy density U_Z permits identification of the photon location z(t) at different times t. Figure 3 shows the variation of photon location z(t) with time t for two values of γ as π and 2π (as series 1 and 2)and it is seen that the velocity $v_g = dz(t)\ /\ dt$ of the photon comes out to be less than the unity value taken for c. By studying other examples with different k_0 and γ values it is found that the photon velocity

fits very well the group velocity formula: $v_g = ck_0 / [k_0^2 + \gamma^2]^{0.5}$
(8)
When the parameter γ is zero , the field solution given by field set (3) can be reduced to that for plane waves , and in that case our value of v_g also becomes equal to c .Thus it is stated that for photons having narrow dimensions in the direction transverse to the direction of propagation, such as for laser beams of light, the velocity of the laser photons in vacuum can be smaller than the standard velocity of light in vacuum, depending upon how narrow is the laser beam of light . For describing the lateral size of the photon in the case of the above studied example ($k_0 = \pi$, $\gamma = \pi$, $\sigma = 0.1\pi$) the energy density u (averaged over the pair of complementary fields) for the n = 0 order is shown with the radial coordinate s in figure 4 at time t = 0 , at position z = 0 ; and at time t = 40 seconds at position $z = v_g t = 28.284$ (as series 1 and 2 respectively) . It is seen that the graphs for the two times almost overlap each other in radial dependence. The transverse radial width of the central peak in the energy density of the photon is found to be nearly $3.832/\gamma$ where the factor 3.832 arises from the first zero of Bessel function J_1 . As an example, for ruby laser light [2] of wavelength 0.6943 micron , with the laser beam radius as 1mm; $k_0 = 9.049669$ micron^{-1} , $\gamma = 3.832$ mm^{-1} , which gives $v_g = 0.9999999103$ c. Thus experimental measurements involving narrow laser beams of light should correct for this reduction in light velocity. It is further noted from figures 2 and 4 that with progression of time , the peak height of the energy density slowly decreases and its width increases along the z axis , due to the individual (k , γ) components in the wave packet having different phase velocities. There is no transverse spreading of the wave packet un-like that for laser beams [2] of light, while the longitudinal spreading is like that for quantum mechanical [3] spreading of particle wave packets. Thus narrow lateral width photons travel at slower than the standard velocity of light, and also have spreading in size, and on both these counts behave like matter particles. If photon energy U is quantized as $\hbar\omega_0$ and photon momentum p as $\hbar k_0$, then equation $(\omega_0/c)^2 = k_0^2 + \gamma^2$ can be transformed to : $U^2 = c^2 p^2 + m_0^2 c^4$,
with $m_0 = \hbar \gamma / c$.
(9)
giving a finite rest mass to the photon depending upon its transverse size, while at the same time providing the relativistic energy-momentum relationship of special theory of relativity [3 & 4]
.

Application to Electrons and Protons:

This analysis is further applied to electrons , considering them to be wave packets of Electro-magnetic field. Taking the rest mass of the electron as m_e and putting it equal to $\hbar \gamma / c$ from equation (9), the value of the Bessel parameter γ is obtained as 2.5896×10^{10} cm^{-1} . Further to consider the electron in motion a typical value of $k_0 = 10000\pi$ is chosen , and σ is taken as 10π for an example. Figure 5 shows the numerically evaluated energy density Uz evaluated from equation (6) as a function of axial coordinate z at times t = 0.0 sec , 0.00001sec , 0.00002 sec and 0.00003 sec . The position of the peak in Uz at time t = 0.00001 second is found to be at z = 0.365 cm , which agrees very well with the group velocity $v_g = 36369.5$ cm/sec given by equation (8) with the chosen values of k_0 and γ. For comparison the plane wave beam of light in time t = 0.00001 sec would have traveled 299792 cm. Thus the electron does behave like a slow photon as described above using electro-magnetic field for its description. Further , an evaluation of particle velocity from $\hbar k_0 / m_e$ also gives its velocity agreeing exactly with that from equation (8) . The radial dependence of the energy density u with radial coordinate s is shown for the n = 0 order considered in figure 6 at time t = 0 sec at z = 0 cm; and at time t = 0.00001 sec at z = 0.365 cm as (series 1 and 2), and it is seen that it falls very rapidly with radius near s = 0 , but at higher s values it falls quite slowly as 1/s , since it is arising due to square of the Bessel functions . The long range slow fall of the energy density u with radius should permit the electron to have long range interaction with other electrons, as is required for Coulomb

interaction[1] between charged particles. This description of the electron as a slow photon allows it to be transversely compact (localized) and still be de-localized enough for long range interactions in transverse directions. Results for motion of protons with time shall be similar to those shown in figures 5 and 6 for the electron, with the mass of the proton m_p to be used in place of mass m_e of the electron in the numerical evaluations.

Linear Momentum:

The linear momentum density [1] of the Electro-magnetic field is given by (**ExB**) / (4π c). Using one TE mode of EM field given by (**TE**)$_1$, and integrating the momentum density over the complete volume , including integration over the range of k values, one obtains the z component of the total momentum of the field (now the particle) as:

$$P_Z = [A^2 / (4\pi c\gamma^2)] . \int (k\omega/c) \, g(k) \, dk . \quad (10)$$

It is easily verified analytically that this result applies equally well for TM and TE modes , for each individual order n of the Bessel function. For the case of non-zero rest mass particles like electron the value of γ is much larger than k_0 and the value of P_Z from equation (10) reduces to

$P_Z = A^2 k_0 / (4\pi c\gamma)$; which is like what it should be for a particle of rest mass $m_0 = \hbar \gamma / c$, rest mass energy as $U_0 = A^2 / (4\pi)$ and velocity $v_g = ck_0 /\gamma$ from equations (7) , (8) and (9) . For the other case where k_0 is much larger than γ , such as for laser beam of light , the value of P_Z from equation (10) reduces to: $P_Z = A^2 [k_0^2 + \sigma^2]/ [4\pi c\gamma^2]$; which on comparison with U from equation (7) shows that $P_Z = U / c$ as it should be for normal photons. Thus our description of **Slow Photons in Vacuum** is capable of explaining several aspects of light as well as of particles having non-zero rest mass like Electrons. A hypotheses is made at this stage of the paper that the **Elementary Particles like Electrons and Protons are actually wave packets of Electro-Magnetic field** and these wave packets need to be further investigated to fully establish the identity of the fundamental particles.

Quantization:

The wave packets are now quantized , so that the amplitude A of the EM fields can be specified, and the quantization condition is defined through the energy of the wave packet , equally well for both TM and TE modes as :

$$U=A^2[k_0^2+\gamma^2+\sigma^2] / [4\pi\gamma^2] = \hbar c [k_0^2+\gamma^2+\sigma^2]^{0.5} . \quad (11)$$

It is seen that in the limit of k_0 much larger than γ and σ , U becomes equal to $\hbar ck_0$ with ω also becoming equal to ck_0 ; and in the limit of γ much larger than k_0 and σ , U becomes equal to $\hbar c\gamma$ with ω also becoming equal to $c\gamma$. The quantization condition (11) thus leads to :

$$A^2 = 4\pi \hbar c \gamma^2 [k_0^2+\gamma^2+\sigma^2]^{-0.5} . \quad (12)$$

The linear momentum in the z direction , under the above given quantization condition becomes:

$$P_Z = \hbar [k_0^2+\gamma^2+\sigma^2]^{-0.5} \int k [k^2+\gamma^2]^{0.5} g(k) \, dk . \quad (13)$$

In the limit of k_0 large compared to γ and σ , P_Z becomes $\hbar k_0$ and U becomes $\hbar c k_0$, leading to P_Z equal to U / c as is normally required for light photons. Further, in the limit of γ large compared to k_0 and σ , P_Z again becomes $\hbar k_0$ while U becomes $\hbar c\gamma$ which is like that for a particle of rest mass $\hbar \gamma / c$ as already noted above .

Angular Momentum:

The EM field set given by the (**TE**)$_1$ mode given by equations (3) for any order n can be used for evaluation of the orbital angular momentum density **r x** (**ExB**) / (4π c) of the EM field for any order n . The z component of the angular momentum density of the EM field , in cylindrical coordinates is s ($E_Z B_S - E_S B_Z$) / (4π c) which for TM modes becomes s E_Z

$B_S / (4\pi c)$ and for TE modes becomes $- s\, E_S\, B_z / (4\pi c)$ for any order n . Integration over the complete volume then gives the z component of Angular Momentum L_z as :

$$L_z = n \left[A^2 / (4\pi c \gamma^2) \right] . \int [k^2+\gamma^2]^{0.5} g(k)\, dk . \quad (14)$$

for each of the $(\mathbf{TE})_1$ and $(\mathbf{TE})_2$ modes, for any order n , with the value of L_z being same for TM modes as well . Upon putting the value of A^2 from equation (12) this value of L_z becomes :

$$L_z = n \hbar [k_0^2+\gamma^2+\sigma^2]^{-0.5} \int [k^2+\gamma^2]^{0.5} g(k)\, dk . \quad (15)$$

It is seen that in the limit of k_0 much larger than γ and σ the L_z becomes equal to $n\hbar$, and in the limit of γ much larger than k_0 and σ the L_z again becomes equal to $n\hbar$. Thus it is seen that for both normal light photons and for electron particle like photon wave packets the z component of angular momentum of the EM field is $n\hbar$. As the Bessel functions J_n can have positive and negative values of n , the z component of angular momentum of any given individual TE (or TM) mode can have a value $n\hbar$ or $-n\hbar$. Thus for a given order of magnitude |n| only two possible values of L_z are allowed as $n\hbar$ or $-n\hbar$, and not (2 n + 1) values . The two spin states of the electron , with spin as $\pm \frac{1}{2}$ as introduced by the description of the electron through the Dirac equation[3] now appear simply as only two possible azimuthal dependences of the wave function, associated with Bessel function of any integer order except for the zeroth order.

It is thus shown that under different values of wave parameters k_0 , γ and Bessel order n these slow EM wave packets in vacuum can appear like light photons as well as like electrons and protons , with regard to their mechanical features of velocity , energy , rest mass , linear momentum and angular momentum.

References
1) J. D .Jackson, "Classical Electrodynamics" IIIrd Edition, Chapter 7,
   John Wiley & Sons, Inc., USA,1999.
2) Ajoy Ghatak and K.Thyagarajan, "Optical Electronics", Chapter 5 and 9 ,
   Cambridge University Press, UK, 1991.
3) Leonard I. Schiff, "Quantum Mechanics" IIIrd Edition, Chapters 3 and 13 ,
   Mc-Graw Hill Book Company, USA,1968.
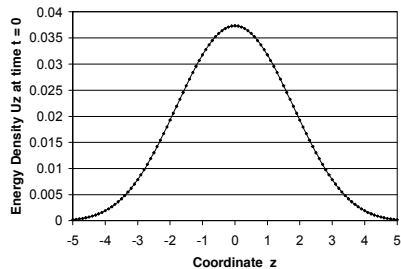4) Albert Einstein, "The Meaning of Relativity" VIth Edition, Chapter 2 ,
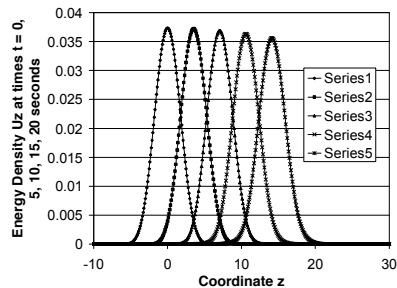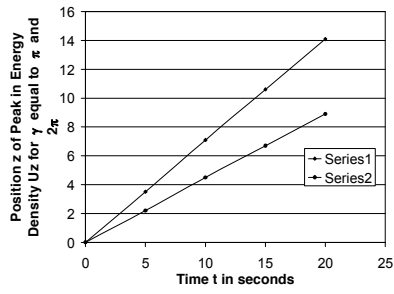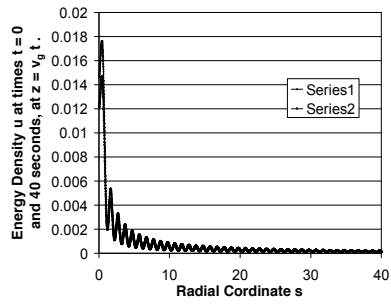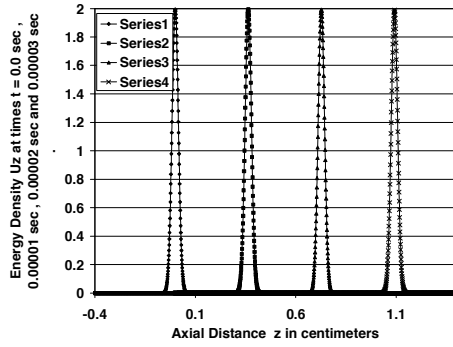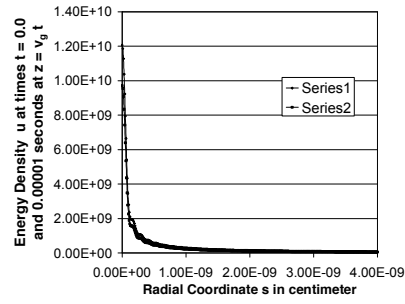   Princeton University Press, USA, 1956.

Figure 1.

Figure 2.

Figure 3.

Figure 4.

Figure 5.

Figure 6.