Mystery And Use of Quantum Optics: How to Teach it to undergraduate Students ?

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ABSTRACT

Photons and laser beams provide an ideal situation to discuss some of the mysteries of quantum physics. There are many additional fundamental features which are not accessible with a classical system such as an electric current, which include: quantum noise, the uncertainty relation and correlations between different laser beams. Not only is the difference between the quantum behaviour of photons and classical waves a curious effect, it is also important in many technical applications and forms the basis of future technologies, such as improved optical sensors or quantum cryptography.

Using the example of a simple beam splitter and its effect on a laser beam one can explore these mysterious quantum effects with undergraduate students. We will discuss a systematic series of cases, using the beam splitter, which identify the difference between the quantum and the classical world. We will present the technical details of an experiment suitable for third year students that demonstrate genuine quantum noise effects.
Keywords: quantum optics, quantum noise, beam splitter, photons, Poissonian statistics, photo-detection

1. INTRODUCTION

Quantum mechanics is a topic which provides many opportunities for motivation and discussions with students. It also is a key concept behind many aspects of modern technology and thus an important building block in the physics teaching program for both engineers and scientists. Optics offers many opportunities to discuss the special features of the quantum world, the difference to the classical world and the resulting mysteries and weirdness.

Visible photons and laser beams are ideally suited to the discussion of quantum effects. One reason is the energy range. For much larger energies per photon, or higher frequencies such as x-rays or gamma-rays, the particle nature completely dominates and the photons show relatively straightforward behaviour. For much smaller energies per photon, or lower frequencies, such as radio waves, the wave nature completely dominates and the quantum effects are not noticeable. In contrast, visible light requires both models and there are many optics applications where we require both models and the technology is moving towards situations where both are of importance.

The first encounter with the concept of photons is normally in the discussion of the creation, or emission, of the light by a light source and the detection, or absorption, of light in a photo detector. Here we refer mainly to the results confirming that the energy of a single photon is linked to the frequency ν of the wave via $E = h\,\nu$. We can discuss the emission of photons by excited atoms and the close link between the differences between energy levels ΔE and the photon energy via $\Delta E = h\,\nu$. At the same time the threshold energy for the photoelectric effect is given by the energy level structure or the band gap of the absorbing material. These simple QM rules are useful but don t lead to many unusual results or mysteries.

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2. SINGLE PHOTON STATISTICS

2.1 A theoretical model

It is getting more interesting when we can consider the Bosonic nature of the photons. The fundamental rules of Boson statistics provide several unusual results different from the classical results. Many of these results are directly linked to the way a beam of photons interact with a partially reflecting mirror, or *beam splitter*.

It is worthwhile to compare the properties of photons and a beam splitter with those of electrons and a junction between two wires. For electrical currents we can apply Kirchoff s law which states that a time varying current $i_{in}(t)$ is divided into two output currents $i_{out1}(t)$ and $i_{out2}(t)$ which, apart from a scaling factor, are identical to each other. That means $i_{out1}(t) = c_1 \; i_{in}(t)$ and $i_{out2}(t) = c_2 \; i_{in}(t)$. The two outputs are perfectly correlated and this includes any signal or noise in the electrical current.

For photons the key observation is that at the input we have a stream of photons, individual particles, which arrive in a certain sequence. The easiest is to assume such a low flux rate that the individual photons are clearly distinguishable. We call this the *single photon regime* in contrast to the *laser beam regime* where the photons follow in such short time intervals that we cannot resolve them. We notice that at the output we again detect photons, the particle is never split into partial photons.

At this stage it is useful to introduce the concept of states. There are several possible states which describe the probability of measuring a photon in a given time interval or not measuring it. These two states can be described by the symbols |1> and |0>. If we had a hypothetical, perfect beam where we have exactly one photon in each time interval we would have the state |1> at the input. We will see below, that in practice that we have |0> for most time intervals and |1> only occasionally. If we concentrate now on only those time intervals where there is a photon we expect to see at the two outputs one beam with |1> and the other with |0> or the reverse. The photon can go to one output *or* the other. It cannot be split and we cannot predict with certainty where it will go. The beam splitter determines the probability of events but not the outcome for any individual photon. We could say that the beam splitter acts like a random selector which sends the photons in either direction, that the randomness of this process is created by the beam splitter.

This has consequences for the statistics of the photon beam. If the input beam was perfectly regular, that means one photon in each time interval, we would have a probability distribution of exactly N_{in} photons detected during each time interval (see Fig. 1). A the output a random selection occurs and we now have a reduction in the average number of photons $N_{out} = t \; N_{in}$ where t is the intensity transmission of the mirror. The width of the distribution function is given by the variance V and is larger for the output than for the input. The output beam is noisy, it has an increased variance. This effect is sometimes called partition noise. Fig.1 illustrates the effect for two cases: Fig.1a for the hypothetical perfectly quiet input beam, which produces two output beams with wider distribution. For a very small transmission, t approaching zero, the output beam will have a *Poissonian* distribution. This is the well known case of selecting a small random sample from the regular input beam and can be derived using combinatorics and binominal distributions. Fig.2a illustrates the case of an input beam with a Poissonian input distribution. Here the two outputs also have a Poissonian distribution which is preserved by partial transmission.

2.2 Practical notes:

Can we produce the perfect beam shown in Fig.1a in the laboratory ? Not yet. We would need a source that can release photons on demand, separated by a fixed time interval and so far there is no device which can achieve this. As a consequence all present laboratory source generate photons in irregular intervals. The best is a Poissonian distribution. This distribution and the underlying noise can be demonstrated using a diode laser with very large attenuation as the source, an amplifier and photon counter as the detector and by making each detection process audible. In this case one hears individual clicks at very low flux and noise, called shot-noise at a higher photon flux.

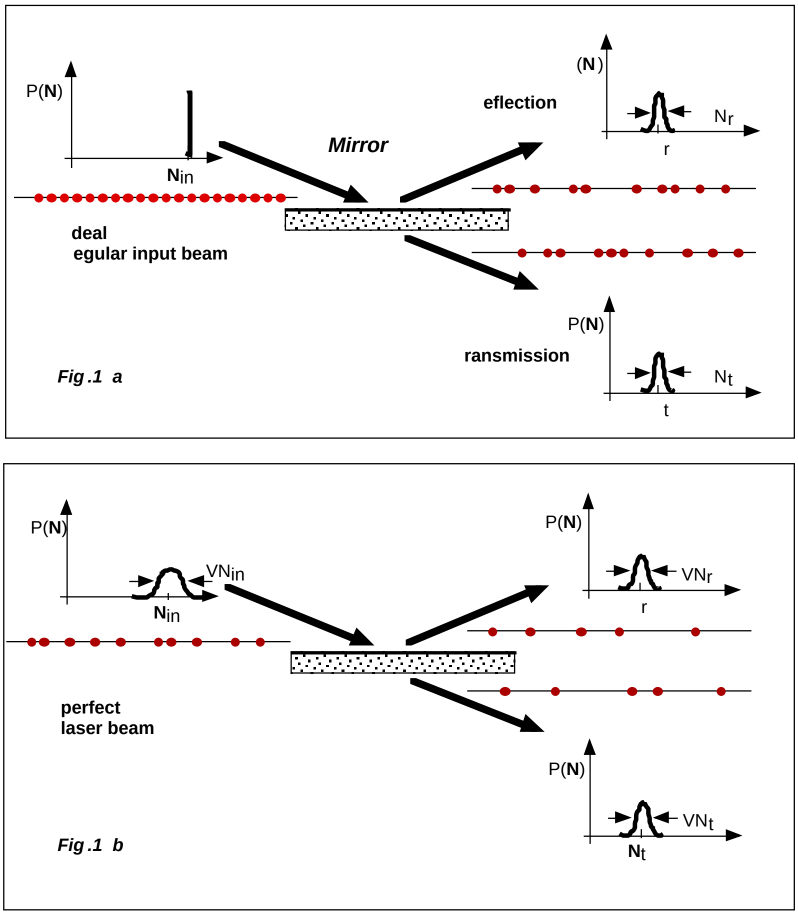

Fig. 1 Illustration of the photon statistics of a beam of photons propagating through a partial mirror, or beam splitter. Individual photons are plotted as small dots. The probability distribution for finding N photons within fixed time intervals are plotted. Case (a) for a hypothetical beam of well regulated photons . Case (b) for the practical case of a beam with Poissonian distribution.

2. LASERBEAMS and LASERNOISE

Any laser will produce at best a beam with Poissonian distribution around an average value N. This is also known in the theoretical literature as a coherent state |a> . The results are that even the best laser will have some residual noise δI in its intensity I. In other words, it is impossible to make a laser which produces a beam intensity that is perfectly quiet. In the best possible case the noise in a beam with average photon number N in a time interval Δt has a standard deviation of photons in Δt of ˆN. That means a relative intensity noise δI/I is given by ˆN. For example, if you have a beam with a flux of 10^{13} photons per second and observe it with a time constant of 1 millisecond you have an average photon number of 10^{10} photons and a relative noise level of 10^{-5}. This noise is called *quantum noise*, since it is a direct consequence of the quantum nature of light. Alternatively it is called shot noise due to its analogy of the noise generated by many pieces of shot falling on a piece of metal. It is also known as white noise, referring to the fact that all frequencies are present simultaneously. It is useful to normalise the intensity noise to this quantum noise limit.

The description of light as a stream of photons is not complete. We also know that the quantum limit for the intensity fluctuations is a direct consequence of the uncertainty principle, and this is a link between two parameters. This is based on a description of the laser beam using *operators* and *expectation values*. This approach requires considerable mathematical skills and we found it not to be suitable to first year university courses. However, a simple version can be used with great effect at later years to discuss the mysteries of QM and its consequences in optics. Such a course would introduce the following concepts: We start with the example of the harmonic oscillator and introduce the operators **a** and **a** . We move on immediately and concentrate on a wave which has a time varying complex amplitude $E(t) = \alpha + dE(t)$. Here a represents the average complex amplitude and can be treated classically, while dE(t) contains all modulation and noise and has to be treated quantum mechanically. That means we have to evaluate the effects of the operators d**a**(t) and d**a** (t). Our measurements can be modelled by expectation values, and essentially there are only two measurement which are performed, namely the average and the variance of an intensity at a photo-detector. The variance is an important concept, it links the measurements we can do with and oscilloscope or spectrum analyser with the quantum mechanics. Once properly understood it allows access to all the experiments done with laser beams.

What are the appropriate operators for a wave. A first guess would be that the corresponding parameters are amplitude E and phase Φ. However, this creates mathematical complications as the phase repeats itself every 2π. The simplest and most elegant approach for the quantisation is to use the operators X1(t) and X2(t) which can be approximated for a large photon flux (N >>1) as

Quadrature amplitude : $X1(t) = E + \delta X1(t) = \mathrm{Real}\,(\alpha) + \delta X1(t)$

Quadrature phase : $X2(t) = \Phi + \delta X2(t) = \mathrm{Im}\,(\alpha) + \delta X2(t)$. (1)

However, in most practical situations we don t deal with the full time history. In order to send information with light we use modulation and create different channels, each around one modulation frequency Ω_{mod}. This is similar to the way we use radio waves; each radio station uses it own frequency and the radio receiver is tuned to only one particular frequency Ω. We can translate the waves described in equ. (1) into a spectrum of the form

$$X1(\Omega) = \mathrm{Real}(\alpha) + \delta X1(\Omega) \quad \text{and} \quad X2\,(\Omega)\_ = \mathrm{Im}\,(\alpha) + \delta X2(\Omega) \qquad (2)$$

where the detection frequency W describes one of the information channels mentioned above. The properties of $\delta X1(\Omega)$) and $\delta X2(\Omega)$ are different from classical modulations. We have distribution functions associated with the two quadrature operators which have the variances $V1(\Omega)$ and $V2(\Omega)$. The uncertainty principle requires $V1(\Omega)*V2(\Omega)=1$. For the ideal laser beam we obtain the symmetric case $V1(\Omega)=V2(\Omega)=1$ which corresponds to the limit of the quantum noise introduced above for streams of photons.

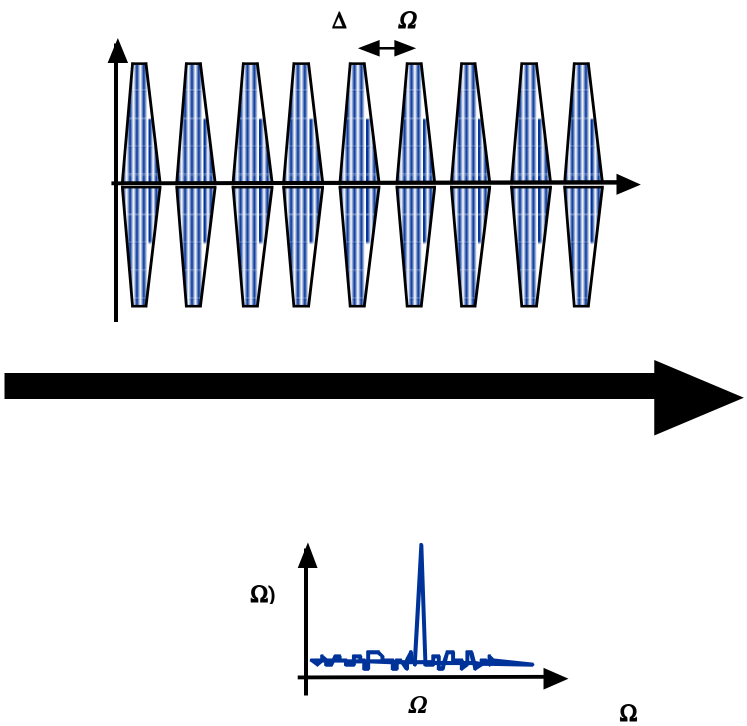

Fig. 2. Link between signal and noise in the time domain and frequency domain. Both representations are linked via the Fourier transform from V1(t) to $V1(\Omega)$ and contain the same information.

3. QUANTUM EFFECTS ON THE QUALITY OF INFORMATION

Information carried in a particular frequency channel which corresponds to V1*signal* and the quality of this signal can be described by (V1*signal* - 1), the distance form the quantum limit. For V1*signal* † 1 the information cannot be distinguished from the noise, the quality is too low. For V1signal >>1 we have a high quality, which growths linear with V1*signal*. In engineering terms this corresponds to the signal to noise ratio. How does the beam splitter affect the quality ? We should calculate V1*signal* at the input and the output of the mirror. The method is to apply the classical model to the operators in equation (2) and to include all inputs to the beam splitter. One input has the signal with $V1_{in1}$= V1*signal*, the other has no light, that means $\alpha = 0$, but the variance is still at the quantum noise limit, that means $V1_{in2} = 1$. (see Fig. 3). This might be a surprise and mysterious, but it is a direct consequence of quantum mechanics: even a light beam with no intensity contains fluctuations. These are the so called zero point fluctuations and with a beam splitter we can observe them directly. The combination of the wave properties and quantisation leads to [2]

$$(V1signal_{r} - 1) = \varepsilon\ (V1signal - 1) \qquad (3)$$

This means that for all situations the quality of the output values is reduced. However, for a very large signal ($V1signal_{in}$ >>1) the effect is small and can be neglected. Once $V1signal_{in}$ approaches the quantum noise the effect is substantial. In summary, we find that for small modulations or strongly attenuated beams the beam splitter has a negative effect, the quality of the signal is significantly reduced. In these situations we have to take the quantum properties of the mirror into account.

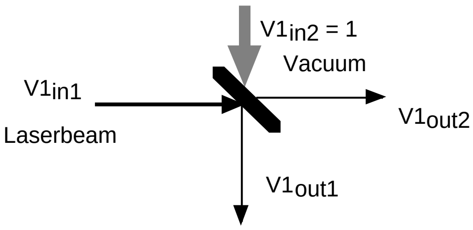

Fig.3 The noise introduced by a mirror can be attributed to the fluctuations of the vacuum beam present at the second input of the beam splitter

The two explanations for the effect of the beam splitter , the photon statistics in section (1) and the vacuum input in section (3) are fully equivalent. The represent the same physics but use different language. These languegaes are actually similar to the use of the Schroedinger picture in the QM of photons and the Heisenberg picture in the QM of continuous laser beams.

In recent years it has been possible to generate light which is even quieter than the best laser beam. Using nonlinear processes it is possible to transfer the noise from one quadrature into the other. For example it is possible to produce a beam with a variance V1 in the amplitude quadrature of less than 1 by propagating a beam through a medium with Kerr nonlinearity. [1]. Since the uncertainty principle dictates V1 * V2 ‡1 the variance V2 of the phase quadrature has to be above the quantum noise limit. Such light is known as squeezed light, the noise in one quadrature is selective suppressed or squeezed away. Equation (3) applies equally well to nonclassical or squeezed light. If we attenuate it with a beam splitter the quality, that means the degree of noise suppression, is reduced. We have to be careful when handling this special light. However, it can also be very useful. If we use squeezed light instead of the normal vacuum at the second port the negative effect of the beam splitter is reduced. With squeezed light we can produce a device that splits a beam and does not degrade the quality. This would have great advantages for eavesdroppers who would like to listen into the information carried by the laser beam . Using perfectly squeezed light they could measure the signal without leaving a trace, a quantum non demolition measurement [4].

4. PRACTICAL DEMONSTRATION OF QUANTUM NOISE

4.1 Experimental layout

It would be very desirable to have an undergraduate experiment which demonstrates the properties of quantum noise to students directly. This could be part of a third year laboratory in physics or photonics. The standard layout of such an experiment is shown in Fig. 3. This arrangement uses an electronic spectrum analyser to record and display directly the spectrum of the variance $V1(\Omega)$ of the intensity noise. In all cases we need a quantum noise limited source for calibration.

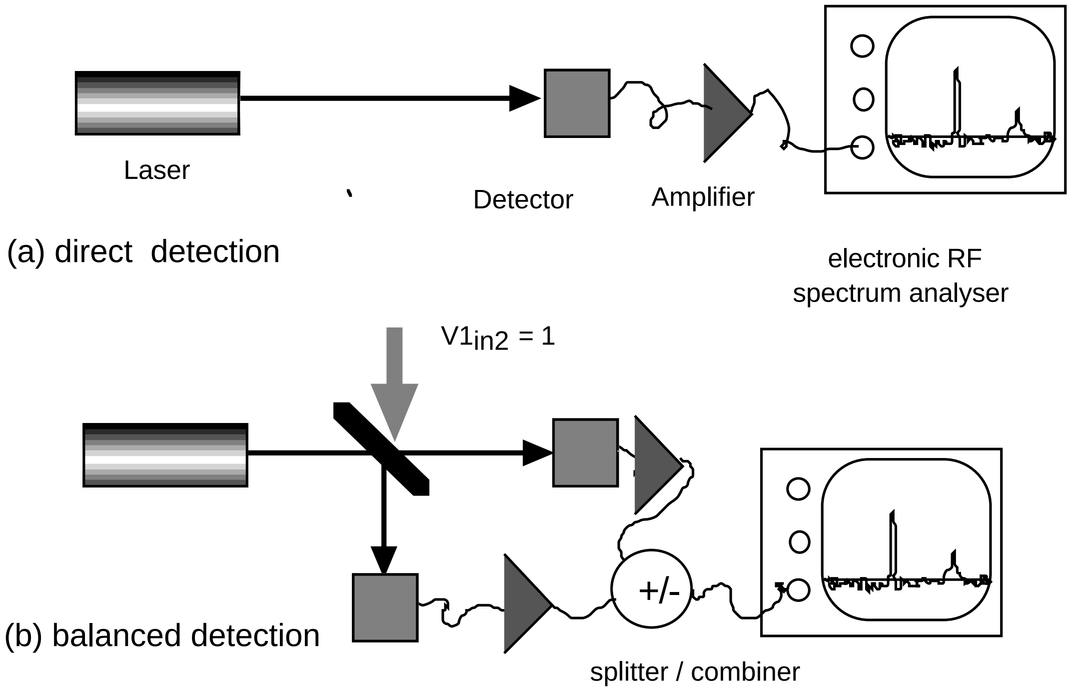

Fig.4 The typical layout for the detection of quantum noise. The scheme in (a) shows the direct detection of one beam with a single detector, followed by an RF amplifier and an electronic RF spectrum analyser. The scheme (b)uses a beam splitter and two detectors and allows the calibration of the quantum noise through the comparison of the sum and difference of the photo currents.

4.2 The equipment. The main components and their specifications are:

(a) A **laser source** which is quantum noise limited in the intensity noise. In most research laboratories the choice are diode pumped solid state lasers, in particular the very low noise diode pumped Nd:YAG monolithic ring oscillators [2]. These laser show a large classical noise signature at about 50 — 500 MHz , depending on pump and threshold power, and are quantum noise limited to better than 2 dB for frequencies of 2 — 20 MHz. Such lasers are very expensive (> 20 kU$). A low cost alternative are diode lasers and He-Ne lasers. The He-Ne Lasers have sufficient power. They operate on several modes, typically separated by 200 — 500 MHz, depending on the length of the resonator and show quantum noise limited performance at frequency between 10 - 20 MHz.

The alternatives are diode lasers. Single mode diode lasers, with external cavity would be perfect, since they have a relaxation oscillation of at least 1 GHz, they have no competing modes and show low noise performance at many frequencies. However, the laser and control unit are still very expensive (>4 kU$). A useful alternative are battery operated laser pointers with a power of 5 mW or larger. Some of these operate in single mode, at a given temperature. Others operate multimode. By selecting the operating temperature and controlling it with a peltier element it is possible to achieve reliable shot noise limited performance. In addition some of these lasers allow direct intensity modulation at practical detection frequencies (1- 20 MHz) and allow a direct comparison between signal and noise (see below).

(b) A photo detector which has a good sensitivity and quantum efficiency, sufficient bandwidth and a low dark-noise. Ideally the detector should have a large quantum efficiency h such that we can see the actual signal and noise on the laser beam. In practice we can achieve $\eta = 0.5 — 0.7$ with silicon photodiodes for wavelengths of 500 — 650 nm or η=0.7—0.8 around 1000 nm with InGaAs photodiodes. The visible range is preferred to make the demonstration experiment more impressive. We need a low noise preamplifiers which has sufficient bandwidth and gain. In our experience the mots suitable are home made amplifiers as described in [5]. We combine this preamplifier with a low noise broadband RF amplifier with a gain of typically 30 dB.

(c) An electronic RF spectrum analyser. This is the most expensive component (> 15kU$). It requires good sensitivity (signal as low as —60dBm) and very low noise floor of about < -90dBm . Almost all RF spectrum analysers have sufficient frequency span (1 - 500 MHz) and resolution bandwidth (10 kHz — 1 MHz). A good detector, amplifier and analyser combination has a low dark-noise, which is tested by simply blocking the detector. In order to perform reliable experiments we require a dark-noise trace which is lower than the quantum noise by 3 dB for optical intensities less than 1 mW and a detection bandwidths of 30 kHz.

4.3The experiment The following experiments provide a good demonstration of the properties of quantum noise:

(1) record the dark-noise and the quantumnoise at about 1 mW power. The actual trace for the dark-noise will be frequency dependent, this is the response function the detector, amplifier and spectrum analyser. The quantum noise should form a spectrum with constant separation from the dark-noise. This indicates the white noise characteristics of the quantum noise.

(2) increase the power from 1 to 2, 4 and 8 mW , if possible. The dark-noise remains unaffected, the quantum noise should rise proportionally to the square of the intensity. This can be observed directly if the dark-noise is more than 6 db below the quantum noise. Otherwise one has to subtract the dark-noise for each recording.

(3) Modulate the laser at a frequency high enough be away from classical noise (typically > 4 MHz) and record the spectrum. This will now have a strong signal at Ωmod with V1(Ωmod) >> V1(Ω) for the neighbouring frequencies. This signal can be seen to stick out of the white noise floor at all frequencies. (see Fig.2). We can assume that in the absence of all classical fluctuations this noise floor is indeed the quantum noise. For a proper calibration see step (5) below.

(4) Now use a 50/50 beam splitter and two detectors. Add the two photo-currents electronically. This requires RF splitter combiner and you have to make certain that the lengths of the cables other delays between the two detectors and the splitter are equal within < 0.1/ Ωmod otherwise you are not recording the true sum and difference. You will find for the sum that signal and noise are the same as if all of the laser beam had been detected on a single detector. Block the beam to one detector and you can show directly how the quality of the signal (the distance between the quantum noise and V1(Ωmod) is reduced. This proofs equation (3).

(5) Use the apparatus from (4) and detect the difference between the photo-currents. We have to trim the gain for the two amplifiers such that the two quantum noise values, detected with the one or the other detector blocked, are equal. Now we are making use of the different correlations between the classical signals at both outputs of the beam splitter. The are fully correlated and the cancel by subtraction and the signal V1(Ωmod) disappears. In contrast there is no such correlation for the quantum noise terms and the quantum noise remains. It will be at the same level as the quantum noise in (4). This result shows that the quantum noise photo-currents are not correlated and that they add in quadrature.

This series of experiments clearly demonstrates the differences between the quantum and the classical model. It becomes very obvious through these demonstrations hat we have to use a full quantum model in any situation where the signal is close in size to the quantum noise level. In modern sensors, which measure external effects such as displacement, stress, temperature, pressure, concentrations etc through the absorption or interference of light, the sensitivity will be limited by this effect. Knowledge about quantum noise will be important for the design of sensitive optical sensors.

5. OUTLOOK INTO THE FUTURE

The TV series Startrek is clearly science fiction. But many of us can see the benefit of teleportation. It would be great for tourists and scientists to hop over to the other side of the globe, or the universe, and it could possibly save energy for the transport. However, until recently we knew perfectly well that teleportation could not be realised. Even if we had found a way of translating a whole living person into pure information we would not be able to send the information to another place and use it for reconstruction since quantum mechanics does not allow the simultaneous measurement and transfer of all the information. This was the situation until 1990 and we dismissed teleportation as violating fundamental rules of Physics. However, this has changed and now it appears that teleportation might be possible at least in principle. Presently the challenge is to demonstrate this new loophole — and again optics is the easiest technology.

The challenge is to transmit all information, in the optical case that means both quadratures X1 and X2 simultaneously. Very much like two simultaneous QND experiments. And the beam splitter plays the crucial role: In order to measure X1 and X2 we need somewhere a beam splitter which sends the information to two detectors. If the second port remains unused we have a degradation of the quality. What could we send into this port? A squeezed beam would allow us to measure one quadrature, as shown above. The trick is to use two squeezed beams which by themselves are mixed on a beam splitter and produce two output beams, which are both very noisy in all quadratures. But they carry identical noise, in the language of quantum mechanics they are entangled. It has been shown, that teleportation is possible if we use these strange beams of light both for measuring and reconstruction of the information. First experimental demonstrations were positive[6].

6. SUMMARY

Optics is a good field to demonstrate quantum effects. The photon model is well suited to show the fundamental differences between a classical model of light and the quantum effects. The first step is to introduce the concept of particles (photons) and the fact that they are indistinguishable. We can use simple mathematical tools, such as combinatorics, to show that light is naturally will contain some noise. A beam splitter is powerful example for introducing the idea that noise is a natural part of light. Perfectly regulated light cannot exist in the presence of losses.

The second step is to introduce the full QM description using operators and states. This is correct for ingle photons and laser beams. We find that for laser beams that can be detected directly with photo diodes a simple and powerful model exits in the form of quadrature operators (X1 and X2) and that the measurement of the variances V1 and V2 includes all practical experiments. Simple rules can be derived for the propagation of the variances and we can model all experiments.

One profound result is the effect of losses on the quality of a signal. Any loss will reduce the quality, which is fundamental to quantum systems in contrast to classical systems. Using an experiment with diode lasers , low noise detectors and an RF spectrum analyser these effect can be demonstrated in the teaching laboratory — directly verifying the quantum theory.

A full teaching course could introduce the future applications of the quantum concept, which include quantum non demolition measurements for tapping information, quantum cryptography, teleportation of optical information and possibly quantum logic using light. These are areas of active research in the moment and most likely practical applications in 10 or 20 years time.

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