

## Quantum Noise, Quantum Measurement, and Squeezing

H. A. Haus

### Introduction

I thank the organizers of this conference for the privilege to present this keynote speech. I am not sure why I deserve this honor. It may be the fact that my first publication on quantum noise, joint with Charles Townes, dates back to 1962,[1] the eventful time when quantum optics was in its infancy.

The talk will be partly an overview, partly a retrospective, finishing with some remarks about the future that will be shaped by the members of this audience. I shall address these topics as I know them best, namely from my own perspective. This approach will not do justice to important players in these developments and hence I want to preface my talk with apologies to all those whose name and work should have been deservedly part of this account.

### Dictionary Definition of Noise

Let me begin with the dictionary definition of noise. The Oxford Universal Dictionary (1955) has the following definition: Noise ...loud outcry, clamour or shouting; din and disturbance; common talk, rumor, evil report, scandal, ... An agreeable or melodious sound. Now rare;...

These are not helpful definitions of the technical meaning of noise. The Oxford English Dictionary (1989) states Noise ...7. In scientific use, a collective term (used with the indefinite article) for: fluctuations or disturbances (usu. irregular) which are not part of the wanted signal, or which interfere with its intelligibility or usefulness.

As you can see, some of the aspects of noise were traditionally not considered unpleasant or obnoxious. The word noise has an etymological connection with music. In German, noise is “Rauschen,” a word with pleasant associations as in Wagner’s composition in his opera Siegfried called Waldrauschen, loosely translated as “forest murmur.”

My own interest in quantum noise arose from my studies of electronic noise in the 1950s. It was evident then that no fundamental limit existed on the noise produced in an electronic amplifier. Shot noise could be reduced by space charge repulsion, now denoted by the more modern term of Coulomb blockade. Thermal noise could be reduced by cooling. With the advent of the laser, operating at frequencies that were four to five orders of magnitude higher than the frequencies of electronic amplification, it became clear that fundamental lower limits on the noise would arise from Heisenberg's uncertainty principle.

### Heisenberg's Uncertainty Principle

Heisenberg's uncertainty principle imposes a limit on the accuracy with which a quantum state can be prepared. For any prepared state, two observables of the state, characterized by their operators  $\hat{A}$  and  $\hat{B}$  must obey the uncertainty product

$$\sqrt{\langle \hat{A}^2 \rangle \langle \Delta \hat{B}^2 \rangle} \geq \frac{1}{2} | [\hat{A}, \hat{B}] | \quad (1)$$

where  $[\hat{A}, \hat{B}]$  is the commutator.

How is this uncertainty product to be interpreted in the context of a measurement? A set of identical states is to be prepared. In one set of measurements, the observable  $\hat{A}$  is measured; in another set of measurements the observable  $\hat{B}$  is measured. When the values of the outcomes of these two sets of measurements are compiled, the mean square deviations of the observed quantities obey (1).

The next question concerns the kind of measurement apparatus used to determine the values of the observables. Idealized von Neumann measurements can serve well. The observable is determined with perfect accuracy, the conjugate observable is rendered completely uncertain.

A photodetector measures the number of photons impinging on it, ideally with perfect accuracy. This means, of course, that the photodetector has to have unity quantum efficiency and zero dark current. No fundamental physical laws prevent an ideal measurement, it can be approached very closely with actual physical devices. However, one cannot claim that the photodetector throws the state of the observable into an eigenstate. The eigenstate would have to be a photon eigenstate. In fact, a photodetector casts the state into a carrier-number eigenstate. The idealized von Neumann measurement is best described in terms of what is now called a quantum non-demolition measurement (QND). I shall describe a QND measurement of photon number later on. Before that, I want to discuss the kind of measurements we normally do; measurements that are aimed at determining observables with the lowest possible quantum noise.

### Simultaneous Measurement of two Non-Commuting Observables

In 1969, Arthurs and Kelly published a paper in the *Bell Systems Technical Journal*.<sup>[2]</sup> They first submitted their paper to *Physical Review*, but it was rejected. In their paper, they studied carefully the implications of a simultaneous measurement of two non-commuting

observables, such as the position and momentum of a particle. They connected to the observable two measurement systems characterized by their Hamiltonians.

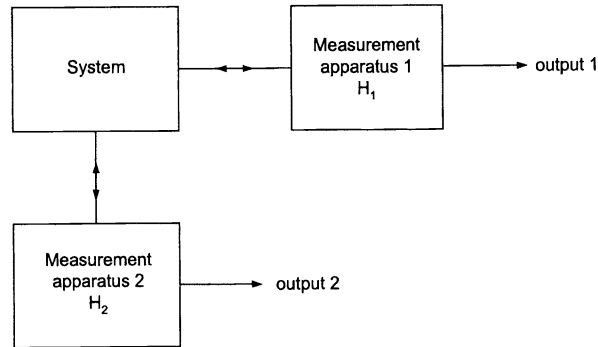


Figure 1: A simultaneous measurement of two observables.

Then, they showed that the additional uncertainties introduced by the measurement apparatus at least doubled the uncertainties of each of the measured observables. This is a major contribution to the theory of quantum measurement, in my opinion. In fact, if a better understanding of the act of measurement had accompanied the development of quantum theory, the “paradoxes” raised in the interpretation of quantum theory would have been more easily resolved. The measurement apparatus designed to yield a result, which can be expressed in “classical” language, *affects the evolution of the state*. To recall the obvious: An electron behaves as either a particle or a wave depending upon the choice of measurement apparatus. The EPR paradox arises from assigning “physical reality” to the quantum evolution of a system. No such assignment is possible without explicit consideration of the measurement apparatus. The spin measurements in the two locations defy causality, if comparisons are made among the outcomes for different settings of the measurement slits, because the measurement apparatus determines the evolution of the combined state of system and apparatus.

This was a brief diversion into the subtleties of the interpretation of quantum mechanics, which many of you could express better than I. Let me describe some aspects and consequences of the Arthurs-Kelly theorem in greater detail. Suppose we make measurements on the states of the electromagnetic field. A coherent state may serve as an example. Figure 2 illustrates the observables  $\hat{A}^{(1)}$  and  $\hat{A}^{(2)}$ .

A classical sinusoid of arbitrary phase serves as the illustration of a coherent state, which may be viewed as the superposition of a cosine and a sine of amplitudes  $\langle \hat{A}^{(1)} \rangle$  and  $\langle \hat{A}^{(2)} \rangle$ . A measurement of  $\hat{A}^{(1)}$  is subject to the uncertainty  $\Delta \hat{A}^{(1)}$ , and a measurement of  $\hat{A}^{(2)}$  is subject to the uncertainty  $\Delta \hat{A}^{(2)}$ . This may be illustrated in the phasor plane by a complex vector with an uncertainty circle; a circle because the process is stationary and phase-independent, and the uncertainties of the in-phase and quadrature components must be equal. The two

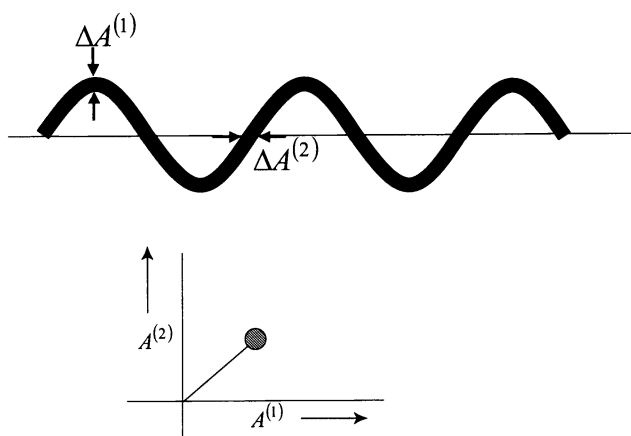


Figure 2: Definition of observables of a coherent state.

operators normalized so that their squares are proportional to the photon number obey the commutation relation

$$[\hat{A}^{(1)}, \hat{A}^{(2)}] = \frac{i}{2} \quad (2)$$

Figure 3 shows a simultaneous measurement of the observables  $\hat{A}^{(1)}$  and  $\hat{A}^{(2)}$ .

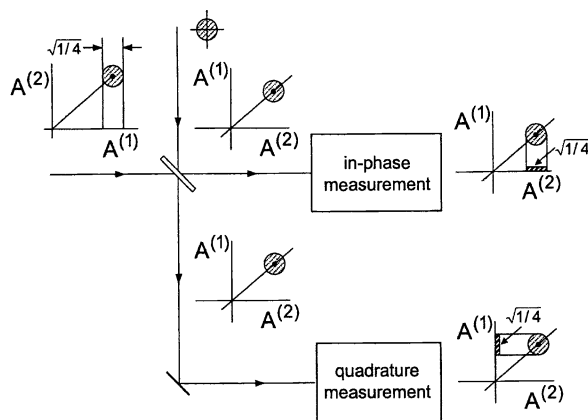


Figure 3: Beam splitter followed by in-phase and quadrature measurement.

The signal phasor is shown in the complex plane and the uncertainty circle indicates that the uncertainties in each are equal, as behooves a coherent state with stationary noise. The signal passes through a 50/50 beam splitter and the two outputs are detected by an in-phase measurement apparatus and a quadrature measurement apparatus. We shall look in detail at a realization of such an apparatus, the balanced homodyne detector. Suffice it to state at this point that each of them can detect the incoming signal with perfect accuracy. This set-up is a simple illustration of the Arthurs-Kelly theorem. Through the vacuum port of the beam splitter enter zero-point fluctuations equal to the fluctuations accompanying

the coherent state. The mean-square fluctuations add at the splitter output, so that full zero-point fluctuations are restored in each of the output ports. The signal power entering each of the measurement apparatuses is halved, the noise is unchanged and, hence, the signal-to-noise ratio is halved and the uncertainty is doubled. This is a clear manifestation of the Arthurs-Kelly theorem. In the apparatus of Figure 3, the idea of a simultaneous measurement is displayed explicitly.

### Balanced Homodyne Detector

Next, we look at the apparatus that makes an in-phase or quadrature measurement possible. Figure 4 shows a balanced detector.

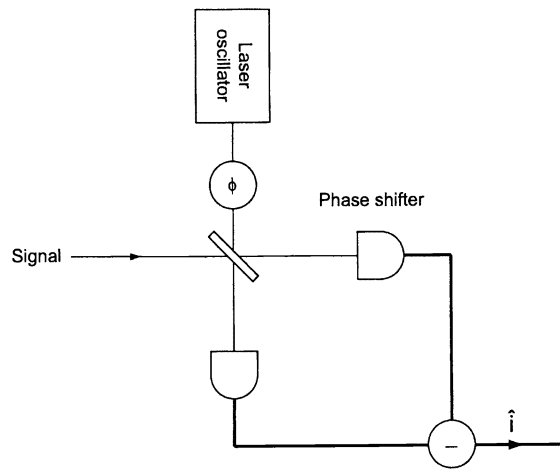


Figure 4: Balanced detector.

Such a detector is an invention that dates back to the days of radar development. The return signal of a radar is 60db or more below the level of the transmitted signal. If the transmitting oscillator were used for the local oscillator in heterodyne detection, with noise that is 30 to 40dB below the transmitter power, this noise would overwhelm the return signal. Hence, it is necessary to suppress the local oscillator noise. This is accomplished with a balanced detector consisting of two detectors whose difference current is taken. Now, the balanced homodyne detector is essential in most quantum-optical measurements. The difference current  $\hat{i}(t)$  of the balanced homodyne detector is:

$$\hat{i} \propto A_{LO}\hat{A}_s^\dagger + A_{LO}^*\hat{A}_s \quad (3)$$

Here  $A_{LO}(t)$  is the local oscillator envelope (the carrier  $\exp(-i\omega_o t)$  has been removed), and  $\hat{A}_s(t)$  is the signal envelope. The local oscillator is treated classically. From (3) one sees that the fluctuations of the local oscillator do not appear, they contribute a term of second order in the fluctuations, which has been dropped. The relative phase between signal and local oscillator determines whether the operator  $\hat{A}_s^{(1)}(t)$  or the operator  $\hat{A}_s^{(2)}(t)$  is measured, with

$$\hat{A}_s^{(1)} = \frac{1}{2} (\hat{A}_s + \hat{A}_s^\dagger) \quad \hat{A}_s^{(2)} = \frac{1}{2i} (\hat{A}_s - \hat{A}_s^\dagger) \quad (4)$$

The local oscillator provides amplification. The measurement of the quadrature component is, effectively, a phase measurement.

We described the measurement of the in-phase and quadrature component of a coherent state using a beam splitter followed by two measurements on the output of the beam splitter. Among the many other possibilities, there is another kind of measurement: amplification to a classical level, which then permits simultaneous measurement without an additional noise penalty. Even though optical amplifiers did not exist in the days when Niels Bohr developed the “world view” of the Copenhagen school, he presaged their measurement capability as the following quote shows:

... every atomic phenomenon is closed in the sense that its observation is based on registrations obtained by means of suitable amplification devices with irreversible functioning such as, for example, permanent marks on a photographic plate ... [3]

### Amplified Spontaneous Emission

A linear optical amplifier of high gain  $G(\gg 1)$  provides at its output a classical version of the input signal with additive noise. The number of noise photons added by amplified spontaneous emission (ASE) in one observation time (inverse bandwidth) is (See Fig. 5)

$$\text{No. of ASE photons} = G - 1 \quad (5)$$

At the output, this stream of photons acts like classical power, with equal in-phase and quadrature (cosine and sine) fluctuations, which are, in photon number units

$$\langle \Delta \hat{A}^{(1)^2} \rangle = \langle \Delta \hat{A}^{(2)^2} \rangle = \frac{1}{2}(G - 1) \quad (6)$$

we find:

$$\frac{\langle \Delta \hat{A}^{(1)^2} \rangle}{G} = \frac{1}{2} \quad \text{for } G \gg 1 \quad (7)$$

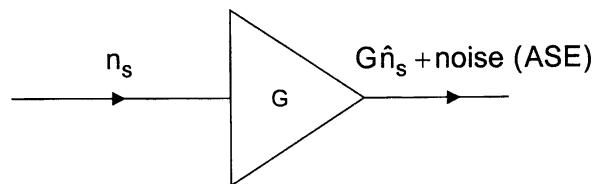


Figure 5: Linear Amplifier.

Note that the fluctuations normalized to the input are double the Heisenberg value, as required by the Arthurs-Kelly theorem on simultaneous measurements of two non-commuting observables.

You may all be familiar with the statement that the minimum noise figure obtainable with an optical amplifier is 3dB. I am currently on the “warpath” against the current definition of noise figure, having given several talks and written several papers on the subject.[4, 5] I advocate a change of definition, so that all frequencies are covered, from radio to optical frequencies, a coverage not possible with the existing definition. I do not want to go into detail here; I only want to mention that the 3-dB limit on the current definition is the consequence of the Arthurs-Kelly theorem.

### Squeezed States

Thus far, I have discussed measurements on coherent states, on states that are stationary: the in-phase and quadrature uncertainties are equal. A preparation of such a state imposes equal uncertainties  $\langle(\Delta\hat{A}^{(1)})^2\rangle$  and  $\langle(\Delta\hat{A}^{(2)})^2\rangle$ . This need not be the case. The measurements to which we paid special attention were simultaneous measurements of two non-commuting observables. If measurements are performed on a single observable, no noise penalty need be incurred. Squeezed states are states that lead to the reduction of noise in the measurement of one observable, and can make the measurement, ideally, noise-free.[6, 7] The best known squeezing process is degenerate parametric amplification. (Fig. 6)

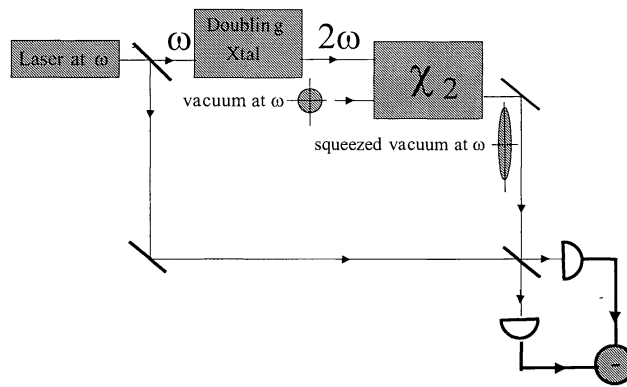


Figure 6: Squeezing with first-order nonlinearity.

A signal at frequency  $\omega$  and a pump at the frequency  $\omega_p = 2\omega$  are passed through a material with first order nonlinearity ( $\chi_2$ ). A special case is when the signal is vacuum. Then the component in one phase with the subharmonic of the pump gets amplified, the component in the quadrature phase gets attenuated. The input vacuum uncertainty circle is transformed into an ellipse of the same area. The fluctuations along the minor axis have been attenuated. Squeezed vacuum injected into a balanced homodyne detector leads to a current noise level below shot noise as can be gathered from (3) when the signal is in the squeezed vacuum state.

Squeezed vacuum is useful for phase measurements below the shot noise level. Figure 7 shows how the noise in such a measurement can be reduced below the shot noise level. Figure 7a describes conventional measurement on an imbalanced Fabry-Perot interferometer. Zero-point fluctuations enter with the probe and through the “unexcited” vacuum port of

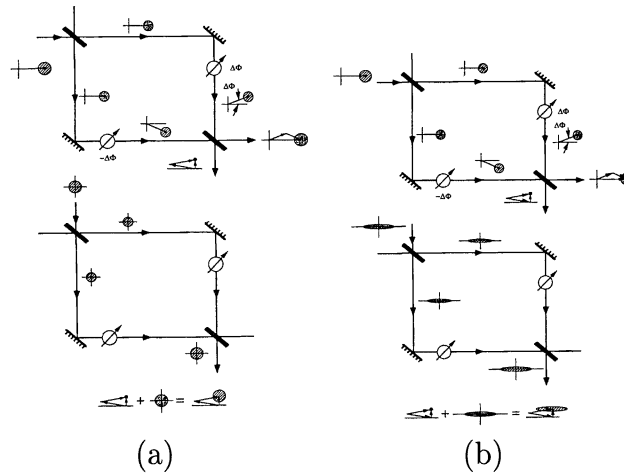


Figure 7: Mach-Zehnder phase measurement (a) standard (b) with squeezed vacuum.

the beam splitter. These zero-point fluctuations add incoherently to their full level and exit through both output ports. If a homodyne measurement is performed on the probe port, they are accompanied by standard zero-point fluctuations.

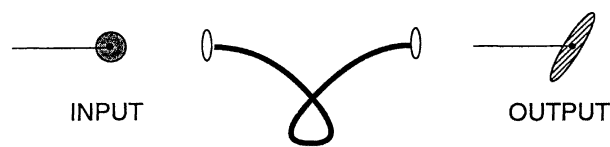
On the other hand, Figure 7b shows the case when squeezed vacuum is injected into the input vacuum port. With proper alignment of the vacuum ellipse, the noise is reduced.

### Squeezing with the Kerr Effect

The first sub-shot noise experiments were performed using the first-order nonlinearity as described. The set-up has the disadvantage that it requires frequency doubling and is difficult to implement with fiber optics. Fibers have a second-order nonlinearity, the Kerr effect, which permits squeezing at the fundamental frequency of the pump. Figure 8 shows how squeezing occurs when a coherent state travels through the fiber. (The parameter  $\kappa$  is a measure of the Kerr effect.) One may view the input as a statistical superposition of phasors of different amplitudes and phases as illustrated by the uncertainty circle. Phasors of different magnitudes experience different phase shifts. To first-order, slices of the input uncertainty circle of different amplitudes slide along each other, deforming the circle into an ellipse.

Following experiments in squeezing at the IBM Almaden Laboratory using the Kerr effect, [8] we developed at MIT a fiber set-up based on a nonlinear interferometer. Figure 9 shows how squeezed vacuum can reduce the shot noise in a balanced homodyne experiment using a Mach-Zehnder interferometer. The input to the Mach-Zehnder interferometer is a sinusoid with accompanying zero-point fluctuations, and vacuum. After the beam splitter, the phasor is reduced by a factor  $1/\sqrt{2}$ , the vacuum fluctuations superimposed from the two sides of the beam splitter are unchanged. The phasors act as pumps in the two nonlinear media. The vacuum is squeezed in the two media. The squeezed vacuums in the two arms are incoherent with each other. After the second beam splitter, the phasor with squeezed fluctuations





$$\langle \Phi \rangle = \kappa L \langle |\hat{a}^\dagger(0)\hat{a}(0)| \rangle$$

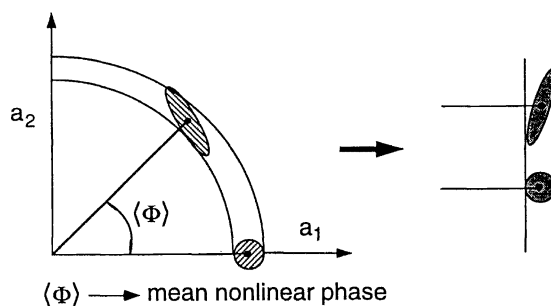


Figure 8: Squeezing in fiber.

emerges from one port and squeezed vacuum from the other port. When the squeezed vacuum is phase shifted so as to align its minor axis along the phasor, then the balanced detector experiences reduced noise. Indeed, in the balanced detector the fluctuations are due to the beat between the noise in phase with the local oscillator, and the local oscillator noise is suppressed to first order.

The operation of a pulse-excited Mach-Zehnder interferometer can be realized with a Sagnac interferometer. The advantage of the Sagnac interferometer is that it is insensitive to environmental changes that are slow compared with the round-trip time through the loop. Figure 10 shows the setup. A mode-locked laser supplies the pump signal. The two pulses formed at the coupler travel in opposite directions around the Sagnac loop.

They exit at the same port from which they entered. Vacuum that entered through the vacuum port and traveled with the two pulses in both directions emerges squeezed from the vacuum port. When the two pulses return to the input coupler, the part of the fluctuations contributed by the noise from the vacuum port exit from the vacuum port, and the noise accompanying the pump leaves through the pump port. The squeezed vacuum is automatically separated from the pump. Experiments at MIT have succeeded in producing a noise reduction of 5.1dB below shot-noise using a pump at  $1.3\mu\text{m}$ . At this wavelength, the fiber has zero group velocity dispersion, different segments of the pulses are independent of each other and squeeze independently of each other. The consequence is that the vacuum ellipses produced by different segments of the pulse differ in ellipticity and orientation (see Figure 11). Upon detection with a detector that integrates over the pulse, all the ellipses

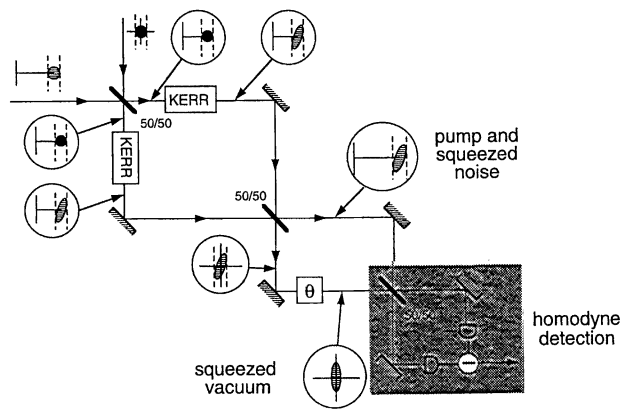


Figure 9: Squeezing in nonlinear Mach-Zehnder interferometer.

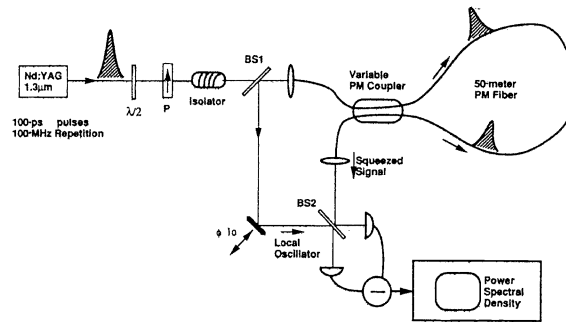


Figure 10: MIT squeezing equipment.[9]

are superimposed. This fact limits the maximum reduction to 7db below shot noise. Thus, 5.1-dB reduction is an impressive result. It should be mentioned that squeezing experiments in fibers are plagued by Guided Acoustic Wave Brillouin Scattering (GAWBS,) discovered by the IBM group. [11] The thermal noise of acoustic waves phase matched to the optical wave can impose noise of their own. Since GAWBS is limited to a frequency range from 10MHz to 500MHZ, use of mode-locked pulses of GHz repetition rate or higher can provide GAWBS-free spectral windows within which the squeezing can be observed.

The squeezed vacuum was used in a phase measurement of an unbalanced Mach-Zehnder interferometer as shown in Figure 12. The squeezed vacuum injected into the vacuum port of a Mach-Zehnder interferometer allowed determination of the phase imbalance 3dB below the shot-noise level (the additional losses incurred in the set-up did not allow the full utilization of the 5.1-dB noise reduction).

### Soliton Squeezing

A pulse propagating in a dispersion-free fiber does not squeeze uniformly across the pulse, thus incurring a limit of 7 dB of maximum noise reduction.[9] A soliton is a collective excitation. The (anomalous) group velocity dispersion, balanced by the Kerr nonlinearity,

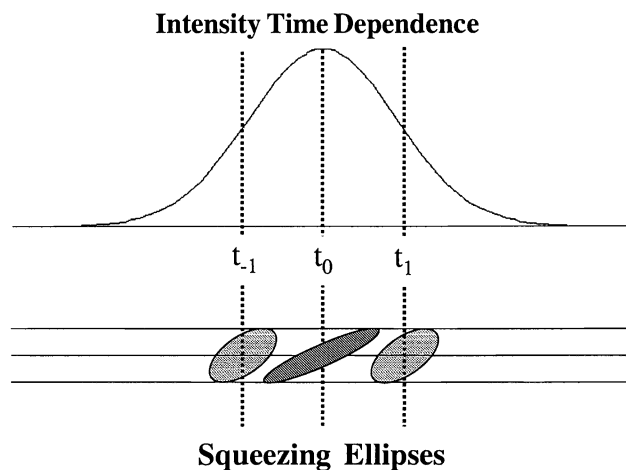


Figure 11: The squeezing ellipses of two different pump intensities.

causes “communication” to occur across the pulse, so that the soliton acts as a single collective excitation. Squeezing of the soliton is a collective phenomenon; the soliton squeezes across its entire profile. A single uncertainty ellipse can be assigned to the soliton as a whole. Therefore, the shot-noise reduction achieved with a soliton is not bound to the 7-dB limit. This fact encouraged us to repeat the squeezing experiment at  $1.55\mu\text{m}$  using solitons as the pump pulses. The set-up is shown in Figure 13. With this setup, 6.1-dB reduction below noise was achieved, a result consistent with the unavoidable losses in the set-up.[12]

### Quantum Nondemoliton Measurement

A photon detector transforms the photons into charged carriers; it “demolishes” the quantity to be measured. A quantum non-demolition (QND) measurement[13, 14]of the photon number, on the other hand, can measure the photon number, cast the state into a photon state and pass the photons on, making multiple measurements on the photon state possible. It is interesting to review the performance of an idealized QND measurement of the photon number.[15] Figure 14 shows the layout.

The apparatus consists of a Mach-Zehnder interferometer. The mirrors  $M_1$  and  $M_2$  are dichroic, they are fully transmitting the signal, but are perfectly reflecting for the probe. A Kerr medium is in one arm of the interferometer. The phase of the probe wave is modulated by the signal. For single photon detection, one would need an unrealistically strong Kerr effect, making the interferometer sensitive to the passage of a single photon. However, even though we do not possess such media, at least now, it is still interesting to explore the workings of the idealized measurement apparatus. We idealize it further by assuming that the Kerr medium is resonant to the beat frequency between signal and probe. Then, the medium does not produce nonlinear phase shifts of the probe alone, or the signal alone. We treat the phase and photon number as two non-commuting observables. The photon number operator and the phase operator obey the commutation relation

$$[\hat{n}, \hat{\phi}] = i \quad (8)$$

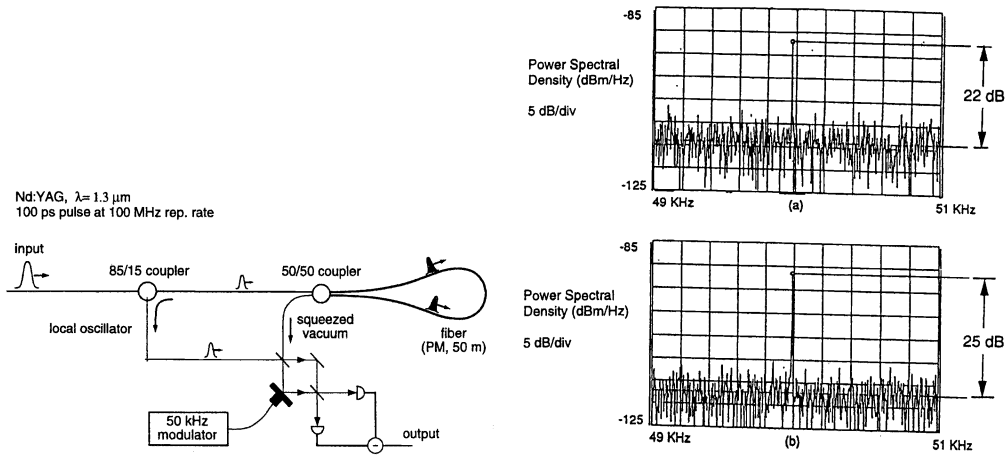


Figure 12: Experimental set-up for measurement of phase:  
 (a) signal and noise with squeezed vacuum blocked, and  
 (b) signal and noise with squeezed vacuum.

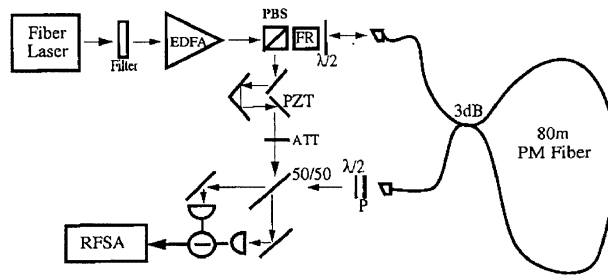


Figure 13: Soliton squeezing.[12]

If the signal and probe phases at input and output are  $\hat{\phi}_s$ ,  $\hat{\phi}'_s$ , and  $\hat{\phi}'_p$ , respectively, then the phase shifts produced in the Kerr medium are

$$\hat{\phi}'_s = \hat{\phi}_s + \kappa \hat{n}_p \quad (9)$$

and

$$\hat{\phi}'_p = \hat{\phi}_p + \kappa \hat{n}_s \quad (10)$$

The phase modulation is a cross-modulation, the signal photon number changes the probe phase and the probe photon number changes the signal phase. The parameter  $\kappa$  characterizes the Kerr medium. The photon numbers are unaffected by the Kerr medium. The signal photon number can be inferred from the probe phase shift in (10):

$$\hat{n}_s^{obs} = \frac{\hat{\phi}'_p - \hat{\phi}_p}{\kappa} \quad (11)$$

The phase difference measured by the Mach-Zehnder interferometer gives information on the photon number. The measurement has uncertainty, in part, due to the inherent uncertainty

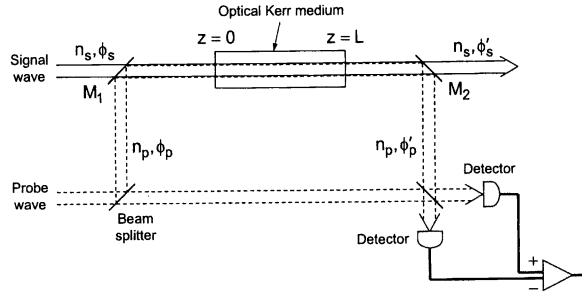


Figure 14: A quantum non-demolition (QND) measurement apparatus[15]

of the photon number and, in part, due to the phase fluctuations of the probe.

$$\langle (\Delta n_s^{obs})^2 \rangle = \langle (\Delta \hat{n}_s)^2 \rangle + \langle (\Delta \phi_p)^2 \rangle / \kappa^2 \quad (12)$$

For a photon number  $\hat{n}_p$  detected by the detector, the phase fluctuations obey the relation

$$\langle (\Delta \hat{\phi}_p)^2 \rangle \langle (\Delta n_p)^2 \rangle \geq \frac{1}{4} \quad (13)$$

If the probe is in a coherent state, then the probe-phase fluctuations obey the uncertainty relation

$$\langle (\Delta \hat{\phi}_p)^2 \rangle = \frac{1}{4} \frac{1}{\langle (\Delta \hat{n}_p)^2 \rangle} = \frac{1}{4 \langle \hat{n}_p \rangle} \quad (14)$$

The mean square uncertainty in the observed photon number is equal to that of the signal plus a term that is made the smaller the more powerful the pump becomes (the larger its photon number), or the larger the Kerr coefficient, or both. In the limit when the second term becomes negligible, the measurement accuracy is perfect. The measured photon number mimics the uncertainty of the incoming signal. If the incoming state is a photon state, then the measured phase difference reveals the number in the state. The photon state remains a photon state. If the incoming state is a coherent state, or any other state with a distribution of photon states, then the knowledge of the phase difference identifies the number of photons in the output state. The output state has become a photon state. As a consequence, the phase of the outgoing signal must be completely uncertain. This follows directly from (9). We have from (9) and (13)

$$\begin{aligned} \langle (\Delta \phi'_s)^2 \rangle &= \langle (\Delta \phi_s)^2 \rangle + \kappa^2 \langle (\Delta \hat{n}_p)^2 \rangle \geq \\ &\langle (\Delta \phi_s)^2 \rangle + \frac{1}{4} \kappa^2 \frac{1}{\langle (\Delta \phi_p)^2 \rangle} \end{aligned} \quad (15)$$

When the error in the measurement is made to approach zero by making  $\frac{\langle (\Delta \phi_p)^2 \rangle}{\kappa^2} \rightarrow 0$ , then the phase uncertainty of the signal after the measurement goes to infinity, as it must for a photon eigenstate. In fact, the QND measurement accomplishes two feats:

1. It generates a photon-number state.
2. It accomplishes squeezing by changing the uncertainty of photon number and phase of the incoming state.

We can cull another interesting feature from the QND measurement of figure 14. We may ask for the uncertainty product of the observed photon number and the phase. A simple computation that is reproduced in the appendix shows that

$$\langle (\Delta \hat{n}_s^{obs})^2 \rangle \langle (\Delta (\phi'_s))^2 \rangle \geq 1 \quad (16)$$

We find that the uncertainty has quadrupled, the Heisenberg uncertainty of each of the factors has doubled. This is the consequence of the Arthurs-Kelly theorem of a simultaneous measurement of two non-commuting observables since the phase of the signal could be measured in a homodyne detector with perfect accuracy.[2]

### Discussion

I have described some specific optical measurements. The simplest and most widely used simultaneous measurement of two non-commuting observables is optical amplification followed by heterodyne detection. The minimum added ASE noise is of the proper magnitude to satisfy the Arthurs-Kelly theorem. The beam splitter, followed by the measurements of in-phase and quadrature components of the incoming coherent wave- packet is another example. The ideal QND measurements using a nonlinear Mach- Zehnder interferometer and detector is another example. When its gain is adjusted so that the photon number is measured with perfect accuracy, then the measurement is of the ideal von Neumann type: the state has been put into a photon number eigenstate. If the gain is relaxed so that the phase is not totally destroyed, the phase can be measured and the set-up again becomes an example of a simultaneous measurement of two non- commuting observables.

The approach I have taken de-mystifies the act of measurement and puts it squarely into the realm of physics. The concept of a quantum measurement was not always this simple. Here is a quote from E.P. Wigner:

...it is the entering of an impression into our consciousness which alters the wave function because it modifies our appraisal of the probabilities for different impressions which we expect to receive in the future.[16]

Such a quote could not escape criticism, as the following quote from J.S. Bell of Bell-inequality fame attests:

Actually, the writers share with most physicists a degree of embarrassment at consciousness being dragged into physics, and share the usual feeling that to consider the universe as a whole is at least immodest, if not blasphemous. However, these are only logical test cases. It seems likely to us that physics will have again

adopted a more objective description of nature long before it begins to understand consciousness, and the universe as a whole may well play no central role in this development.[17]

The approach I presented “plants its feet” squarely in the Bell camp. I have insisted that the concept of a measurement cannot be understood properly until the act of measurement is properly described with the specifics of the measurement apparatus clearly stated.

I have mentioned in the introduction that my interest in quantum optics dates back into the early 1960s. In my work I have concentrated on quantum optics involving many photons, because I believed that engineering applications call for states involving low quantum noise, high SNRs, so as to permit rapid reception. Researchers like the late Professor Mandel of Rochester [18, 19, 20] and others have pursued the generation and measurement of states involving few photons, all the way to the generation and detection of entangled states of single photons. Quantum mechanics, in all its subtleties is revealed in the behavior of these states. Aspect and his coworkers [21, 22, 23] have proven the violation of Bell’s inequality using polarized photon states. Quantum encryption, quantum computing, quantum teleportation are now the key words for intensive research in quantum optics. This work is exciting and will reward us with an even better understanding of the theory of quantum measurement. Will it lead to practical applications? Predictions are difficult, particularly predictions of the future, as someone has said. Challenges are more predictable. Quantum optics is “analog” rather than digital. The vulnerability of “analog” signal processing will have to be confronted with clever “forward error correction” codes, or the like of it.[24] Quantum teleportation is slow compared to existing transmission speeds of 40Gbit/s. It has to perform functions that are superior to those of conventional transmission of today and the future, when competitive classical schemes are likely to be developed. These are exciting challenges. Many of you will face them, some of you will overcome them. I wish you all full enjoyment of the work ahead.

# Appendix

## Proof of Inequality (14)

We now derive the uncertainty product of the observed photon-number fluctuations and the phase imparted to the state emitted by the apparatus. We have from (12) and (13)

$$\begin{aligned} \langle (\Delta \hat{n}_s^{obs})^2 \rangle \langle (\Delta \hat{\phi}'_s)^2 \rangle &\geq \left[ \langle (\Delta \hat{n}_s)^2 \rangle + \langle (\Delta \hat{\phi}_p)^2 \rangle / \kappa^2 \left[ \langle (\Delta \phi_s)^2 \rangle + \frac{1}{4} \frac{\kappa^2}{\langle (\Delta \hat{\phi}_p^2) \rangle} \right] \right] \\ &\geq \frac{1}{2} + \frac{1}{4} \left[ \frac{4 \langle (\Delta \hat{\phi}_s)^2 \rangle \langle (\Delta \hat{\phi}_p)^2 \rangle}{\kappa^2} + \frac{\kappa^2}{4 \langle (\Delta \hat{\phi}_s)^2 \rangle \langle (\Delta \hat{\phi}_p)^2 \rangle} \right] \geq 1 \end{aligned} \tag{A.1}$$

We have assumed that both signal and pump are in a coherent state. The equality sign is realized when the system is adjusted so that

$$\frac{4 \langle (\Delta \hat{\phi}_s)^2 \rangle \langle (\Delta \hat{\phi}_p)^2 \rangle}{\kappa^2} = 1 \tag{A.2}$$



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