A theory of quantum dynamics of a nanomagnet under excitation

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ABSTRACT

A quantum treatment of magnetization dynamics of a nanomagnet between a thousand and a million spins may be needed as the magnet interacts with quantum control. The advantage of the all-quantum approach over the classical treatment of magnetization is the accounting for the correlation between the magnet and the control agent and the first-principles source of noise. This supplement to the conference talk will concentrate on an overview of the theory with a presentation of the basic ideas which could have wide applications and illustrations with some results. Details of applications to specific models are or will be published elsewhere. A clear concept of the structure of the ground and excited macrospin states as magnetization rotation states and magnons in the Bloch/Dyson sense gives rise to a consistent theory of the magnetization dynamics of a ferromagnet modeled by the Heisenberg Hamiltonian. An example of quantum control is the spin torque transfer, treated here as a sequence of scatterings of each current electron with the localized electrons of the ferromagnet, yields in each encounter a probability distribution of the magnetization recoil state correlated with each outgoing state of the electron. This picture provides a natural Monte Carlo process for simulation of the dynamics in which the probability is determined by quantum mechanics. The computed results of mean motion, noise and damping of the magnetization will be discussed.

Keywords: magnetization dynamics, nanomagnet, spin transfer torque, magnons, spin waves, micromagnetics, spintronics

1. INTRODUCTION

Dynamics of magnetization of a ferromagnet is both fundamental to the physics of magnetism and indispensable to the application of spintronics. The prevalent theory and computation methods are based on the "semiclassical theory". By semiclassical, it is meant here as treating the magnetization dynamics as classical always and treating the driving mechanisms as quantum or classical as appropriate.[1,2] For example, in the spin-polarized current driven magnetization dynamics for the spin transfer torque (STT),[3,4] the current electron is treated quantum mechanically and the magnetization classically. In optical control, the light is classical but the dynamics of the optically excited electron which drives the magnetization is quantum mechanical.[5] The semiclassical theory for STT has been wildly successful (see the collection of articles on the 2008 perspectives, in particular the introduction[6] and the simulations[7,8]).

So, what is the point of an all-quantum approach? The quantum dynamics of the magnetization is defined as the dynamics of the collective states of the localized spins which constitute the magnetization. This notion can be used for magnetic insulators and metals. The motivation for including the magnetization states as quantum stems from the following list of questions:

1. As the size of the magnet or its number of spins is reduced, when does the crossover from the classical to the quantum regime occur?
2. In the semiclassical theory, the dynamics of the quantum objects, be them electrons, magnons, or phonons, treats the magnetization in the mean-field approximation. Thus, the quantum correlated dynamics between the quantum particles and the magnetization states is removed. What are the observable properties which would distinguish the two treatments of the magnet?

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3. There are two types of measurements. In one, the driving agent is measured, such as the current behavior in STT and the magnetization reversal is inferred. The other is the direct magnetization measurement. Will the combination of the two types of measurement be able to test quantitatively semiclassical theory versus all-quantum theory?

2. NATURE OF SEMICLASSICAL AND QUANTUM APPROACHES

Here is a perspective on possible ways to answer the questions above which frames the outline of this talk.

2.1 Size Range of the Magnet

The size of the nanomagnets measured in the total number of spins ranges from the micron range from 10^{10} to 10^8 through the nano range of $10^7 - 10^3$ to the molecular magnets of 10^2 to 10 spins, with, respectively, the macroscopic, mesoscopic and microscopic range of properties, as classified by Werndorfer.[9] The mesoscopic range is the topic of interest here. This is the range where the crossover from classical to quantum behavior of the magnetization dynamics is most likely to be found. The size range spanned by the nanomagnets corresponds at the high end to the samples such as the nanopillars used in experiments[10] and at the low end to the current attempt at fabrication of bit patterned media[11] for memory device of terabits/in^2. The study of the dynamical behavior of the smallest possible nanomagnet is, of course, of interest for the increase of memory density but it is also the basis as the cell unit for the classical depiction of magnetization dynamics.[1,2] Further study in the quantum approach beyond the justification of the Hamiltonian for the Landau-Liftschitz approximation may be of benefits to the semiclassical computations as well. We have thus defined the arena for seeking the answer to Question 1 in Sec. 1. One method to study the crossover is the comparative study of the semiclassical method and the quantum method for selected properties of a few key models across the mesoscopic range.

2.2 Basic Ingredients of a Quantum Approach

A possible quantum approach may start with a single domain study. To facilitate comparison of results with the semiclassical approach, the approach may start with a spin Hamiltonian with the Heisenberg exchange interaction between pairs of spins, anisotropy from the spin pair interaction whose total energy agrees with the classical anisotropy energy as a function of the magnetization, and the quantized version of the magnetostatic terms. Since the equation of motion for the classical magnetization is derived from consideration of such a Hamiltonian,[1,2] there should be no question of validity of the Hamiltonian for the more microscopic approach. It is generally recognized in this field that the Heisenberg model is not ideal but acceptable for ferromagnetic metals as well. Once the ramifications of this model is well understood, one may then investigate the more advanced models based on a hybridization of mobile and local electrons.[1,2,12]

The eigenstates of the Hamiltonian form the basis set for the magnetization state. The state is generally mixed, i.e., characterized by a probability distribution of quantum states, thus, by a density matrix. What is known in the classical theory as magnetization is equivalent to the expectation value of the magnetization operator with respective to the density matrix. The magnetization dynamics is derived from either the time dependence of the magnet state or of the magnetization operator. While this may be a complex many-body problem, in this, the 21st, century we have the tools for an attempt at both the analytic and numerical studies (actually indivisible twins) with guidance by the experiments and the physical understandings acquired so far. Some examples will be given below but, regrettably, proof is absent. As someone once said, "a demonstration may satisfy the reasonable person but a proof is necessary for the obstinate."

The macro-spin states refer to the set of states of maximum magnitude of the total spin (defined precisely in Sec. 3). They form the set of states for all possible rotation of the state with all the spins pointing in one direction, any direction. These are the quantum counterpart of the magnetization in the semiclassical Landau-Liftschitz theory. The excited states, actually the collective states of less total spin, are commonly regarded as wave-like deviations of the local spins from the magnetization direction. The additional unit of energy to the macro-spin ground state is known as an elementary particle named magnon.

When the perturbations to the above Hamiltonian provide transitions between the macro-spin states and the magnon states, these rather dense (almost continuum) magnon states provide a source of decoherence and dissipation of the macro-spin states. The perturbations can be impurity spins, surface defects, spin-orbit interaction, spin-orbit assisted phonon interaction and hyperfine interaction with the nuclear spins. Such studies in theory and experiment can pinpoint the physical origin of the phenomenological damping parameters in the semiclassical theory and provide comparison between the all-quantum theory with the semiclassical theory in dealing with the magnetization and magnon interaction.

The difference between the semiclassical approach and the quantum approach that is claimed in the introduction to be the quantum correlation is easily illustrated by the spin torque transfer.[13] In the quantum scattering between the current electrons and the magnet spins, the electrons in the current may be considered to be independent of one another. In the oversimplified picture, after a current electron scatters with all the spins of the magnet, each of the set of outgoing spin states of the electron is associated with a collective spin state of the magnet. Thus, the state of the system of the current electron and the entire magnet is an entangled state in which a magnet state is conditioned on each outgoing spin state of the current electron. It is highly improbable that all the magnet states are identical. This is the consequence of the quantum correlation missing in the semiclassical treatment.

What are the effects of this quantum correlation? In the limit of large magnets, it can be shown that the quantum solution leads to the mean semiclassical motion.[14] It would be interesting to find how small the magnet would have to be for the difference to be manifest. The fundamental difference in the dynamics may be described for two different conditions of the current electrons:

1. For independent electrons, a second electron hitting the magnet would find a probability distribution of the magnet states conditioned on the outgoing state of the first electron. This leads to a natural stochastic process in the spin torque transfer in which the probability is governed by the quantum scattering between the current and the magnet. For simulation, this leads to a Monte Carlo process in which the probability is given by quantum dice.[13] The STT is the recoil from the current-magnet collision. It does not appeal to conservation of angular momentum which equates the gain in magnet angular momentum to the loss in current angular momentum scattered by a stationary magnetization, as in the semiclassical method.

2. When the current electrons are coherent, interesting effects such as stronger STT than the normal current case might be expected. Two pulses of current coherent with each other have been used to generate stronger STT.[15] The ability to control coherent pulses shows the potential of control of STT to be stronger or weaker. This is just due to the coherence of two envelopes of current pulses. Phase lock of the central frequency of two pulses might bring even stronger coherence effect of the magnetization dynamics. In this sense, the effect of superconducting current on STT[16] should be of great interest even though this is a low temperature phenomena, unsuitable for practical memory devices. But the interest in superconductor/ferromagnet has been mostly on the Andreev reflection as a probe[17] or the proximity effect.[18]

Magnetization fluctuation provides a particularly clear distinction between the semiclassical and quantum approach. It is also of key importance for the spintronics' future in that fluctuations play an important role in superparamagnetism on which the magnetic dots rely. Fluctuation determines the signal-to-noise ratio, a key factor in the device performance.[19] The classical approach to magnetization fluctuation, as pioneered by Brown,[20] relies on grafting a fluctuation dynamic variable to the classical equation of motion for the magnetization, leading to a Langevin equation in non-equilibrium statistical mechanics. A stochastic assumption is made for the dynamics by inserting a random variable in the driving term, which has a dissipation term and a probability distribution with the Markovian approximation, which leads to the Fokker-Planck equation for the distribution.[21,22] The semiclassical part comes from treating the current electrons in STT as quantum scattering against the stochastic magnetization.[22–25]

By contrast, quantum dynamics naturally produces all fluctuations. The source of fluctuation completely absent from the semiclassical approach is that arising from the entangled states between the control and the magnet. In the STT example, the current electron scattering with the magnetization produces a probability distribution of the magnetization states conditioned on the current electron states and vice versa. The individual scatterings of the current electrons produce shot noise in the current and shot noise in the magnet motion. The experimental implication will be discussed next. The scattering states of both control and magnet constitute also the conduit of passing native fluctuations of one to the other, such as the Johnson noise of the current to the magnet motion and the fluctuations from magnons and macro-spin states to the current. Just as the spin torque transfer itself, the fluctuations come not from conservation rules but from the dynamics itself. The fluctuations will be discussed in terms of the results of model computations in Sec. 3.

2.3 Relevant Experiments

The experiments we discuss here are not the measurements of properties of the magnetic system, which are essential for theory, but the ones from which one can infer the single magnetization dynamics and its back-action on the driving system, such as on the STT current in a spin valve. In principle, consistent measurements both on magnet and on control may provide a means to test any differences in the semiclassical and quantum approaches.

Interesting nonlinear oscillations of the magnetization have been given a thorough theoretical study in the microwave range,[26] whose predictions have been confirmed by experiments.[27,28] An earlier review[29] of the STT nano-oscillators has shown that the search for higher frequency than 10 GHz and narrow linewidth may involve nonuniform magnetization. This might be grounds for the quantum effects of the magnetic states. Recent development in fast time-resolved measurements[30–32] and coherent control[15] have made possible studies of magnetization dynamics and fluctuations which may need a quantum treatment of the magnetization dynamics.

In semiconductors, spin polarization in currents is measured by spatial and time resolved Kerr microscopy and by the Hanler effect, namely the electron spin polarization induced dynamic nuclear polarization spectroscopy.[33,34] The quantum prediction[35] of the spin accumulation by the spin reflection of current in the paramagnetic semiconductor from scattering against the ferromagnet speaks against the naive picture of the origin of spin current.[36]

The experiments described above are current measurements, which are the main method in deducing the magnetization dynamics.[6] While there were direct measurements of reversal by magnetic force microscopy[37] and by scanning tunneling microscopy,[38] the reversal was not STT driven. The recent progress in direct time-resolved measurements of the magnetization dynamics by X-ray[39,40] are used or may well be used in conjunction with the measurements of the driving system. XANES (x-ray absorption near edge structure) has been used for studies of high spin states.[41] Might its capability in femtosecond time-resolved measurement and polarization discrimination not be developed for direct measurement of macro-spin states and magnon states in a small nano magnet?

Finally, the shot noise measurements[42,43] in STT open up the question whether it is due to the atomic nature of the magnetization recoil from scattering or from that of the current through the tunnel junction not related to the magnetization dynamics.[44] This will be discussed in Sec. 3.1.1.

3. QUANTUM MAGNETIZATION DYNAMICS

Quantum dynamics of magnetization is based on the state representation of all the spins in the ferromagnet. We start with a description of the basis states convenient for conceptualization and computation.

The exchange energy is by far the largest term in the Hamiltonian described in Sec. 2.2. In the Heisenberg part which has spherical symmetry, for a ferromagnet of N spins, the degenerate ground states are denoted by $|J, J-m\rangle$, where $J = N/2$ and $m = 0, 1, \ldots, 2J$. They may be viewed as the macro-spin states which form the basis for the rotation of the maximum magnetization of length J. A magnetic field would split the degenerate states into a ladder. A uniaxial anisotropy energy between two spins (about 1/10 of the pair exchange energy) would split the degenerate ground states into a ladder with rungs rising and falling from $m = 0$ to $2N$.

For a ferromagnet, the lowest energy state has the full magnetization, represented by the state $|J, M = J\rangle$ with the spin axis along the magnetization, where J is the spin quantum number and M is the magnetic number along the spin axis. When the magnetization state is rotated, the state becomes a coherent combination of all the $|J, M\rangle$, known as the spin coherent state,[45] as the rotationally displaced state analogous to the coherent state from the linearly displaced harmonic oscillator ground state. These states are known as the macrospin states, all having the same maximum magnitude. Besides these states, the excited states may be taken in the bases of $|J-j, J-m, r\rangle$ states, where the integer values j decreases $J-j$ to 0 or 1/2 and the additional index

r denotes the different irreducible representations of the states $J-j, J-m$. For a lattice of spins with periodic boundary conditions, r may be products of wave vectors. The states $|J-j, J-m, r\rangle$ are then m-magnon or spin wave states. We will refer to these states as m-magnon states even when there is no lattice periodicity. Note that this approach follows that of Bloch,[46] Dyson[47] and others.[48,49] The much used Holstein-Primakoff[50] is equivalent to the Bloch-Dyson theory at the one-magnon level, valid for small angle rotations but is inaccurate in magnon-magnon interactions.[47]

Below is a progress report on what we have computed within the all-quantum approach using simple models for the STT on macrospins and on adding magnon states.

3.1 A Model of Macro-spin States

3.1.1 Current driven macro-spin dynamics

The discussion here of the first paper by Wang and Sham[13] will focus on the physics of the quantum dynamics of the macro-spin states driven by STT. Quantitative details will be referred to the paper. To concentrate on the macro-spin dynamics driven by a spin polarized current, we start with a simplification of the Hamiltonian described in Sec. 2.2 by making the position dependence of the magnet spins homogeneous like a jellium. Thus, the Hamiltonian is given by,

$$\hat{H} = \left(-\frac{1}{2}\nabla^2\right)\hat{s}_0\hat{\mathrm{J}}_0 + \delta(x)\left(\lambda_0\ \hat{s}_0\hat{\mathrm{J}}_0 + \lambda\ \hat{\mathbf{s}}\cdot\hat{\mathbf{J}}\right). \quad (1)$$

The first term is the kinetic energy of the current electron. The spin independent terms are represented by the unit operator $\hat{J}_0$ for the macrospin space and $\hat{s}_0$ for the current electron spin space. The total spin $\hat{\mathbf{J}} = \sum_l \hat{s}_l$, which is the sum of spins located at sites l, is presented to the incoming current spin $\hat{s}$ as independent of the local spin position. The spin-independent potential λ_0 is needed to preserve some important spin effects which could be eliminated by having only the λ term. This simplified model Hamiltonian enables an analytic expression of the scattering matrix between the two dynamic systems - current electron and the slab of ferromagnet. The defect of the delta function model is easily removed by the more general S matrix elements.

Two Cartesian frames are set up, one for the current electron spin and the other for the macro-spin, the latter being termed the Berger frame.[4] In the former, the x-axis is normal to the slab and the z direction to be the spin up state $|+\rangle$ of the incoming current electron. For the macro-spin state, the the spin up axis for all the magnet spins is along the initial magnetization orientation. The orientation of the macro-spin state in general is denoted by the polar angles Θ, Φ. For an incoming electron with wave vector k perpendicular to the interface, the scattering is given by the transformation of the composite macro-spin and current spin state,

$$|J,\Theta=0,\Phi=0\rangle|k,+\rangle \rightarrow g_1|-k,+\rangle|J,\Theta_r,\Phi_r\rangle + g_2|k,+\rangle|J,\Theta_t,\Phi_t\rangle + g_3|-k,-\rangle|J,0,0\rangle + g_4|k,-\rangle|J,0,0\rangle, \quad (2)$$

where the initial macro-spin orientation, $(\Theta = 0, \Phi = 0)$, is taken in computation to be about 0.5 radian off the anti-alignment with the current spin polarization direction $z+$, the suffix r denotes the association with the reflected electron state, and t the transmitted. The last two terms reflect the outgoing macro-spin states unaffected by the spin flip terms.

A stochastic trajectory of the macro-spin state is determined by the Monte Carlo selection of each sequential outing state given by probability distribution for each incoming electron given by scattering, $|g_i|^2$. The number of magnet spins range from 10^6 to 10^4. The number of current electrons of the order 10^7 is sufficient to make the driving duration to cover the magnetization reversal. A sample of the order 10^3 trajectories yields the mean magnetization dynamics similar to the semiclassical simulations, with the almost one complete cycle of precession about the z axis of the current electron and reversal in time of the order of nanoseconds with the electron velocity of the order of 10^6 m/s, in the ballpark of the experimental values. The similarity with the semiclassical results in mean dynamics is not surprising. It would take more accurate models and more detailed computation to find the difference between the two approaches. The fluctuation of the path is, however, absent in the semiclassical approach. The uncertainty calculated of the rotational path results from the scattering is significant in the case of 10^4 spins. While the uncertainty is of order $1/\sqrt{N}$ for N spins, more important is how this would affect the stability of the magnetic bit for small nanomagnets and sensitive measurements of only part of a nanomagnet.

Computation was made for noise of the magnetization rotation from each of the four sources: (i) the entanglement of outgoing current electron states with the macro-spin states - the basic STT noise; (ii) the noise due to the Fermi distribution of the momentum of the current electrons; (iii) the spin noise which reduces the spin polarization of the current; (iv) thermal distribution of the initial macro-spin state. The scattering and the spin sources are of comparable importance, much stronger than the momentum distribution. The noise was calculated for both the current and the magnetization, which provides a possibility for experimental test of the premise of the connection through the scattering entanglement as the conduit for all three types of noise. For 10^4 spins, the uncertainties of the magnetization components could be amplified several times the initial thermal uncertainty. The peak uncertainty is inversely proportional to the number of spins and the time it takes to teach the peak (about half the switching time).

Current shot noise under STT through an MgO tunnel junction has been measured below 5K.[42] Our computed values have same dependence on spin valve magnet alignment (about a factor of 4–5 higher for antiparallel than parallel configuration) but a factor of 3–4 less than the measured values. There is a simple explanation of the configuration dependence from the quantum approach (see Sec. 3.1.2). The discrepancy in magnitude may be due to the weaker materials parameter used for computation (Fe) and for experiment (CoFeB). The experimentalists[42] inferred the current noise to be of magnetic origin. Our computation is based on the scattering noise. Another calculation[51] attributed the origin to the interface disorder affecting the MgO tunnel junction. Since this theory did not include the effect of the magnetization recoil and chose a particular value of disorder to fit the agreement with experiment, further work including both effects are needed to discern whether either disorder or recoil dominates or both.

3.1.2 The open system approach

An illustration is given to solve the macro-spin model as an open system dynamics of the macro-spin subject to the bath of the current electrons,[14] using the theory developed for quantum optics.[52] Later the magnon states are added as a second bath. In fact, by treating all three elements as the same quantum system, there is a possibility that the control (the current) and the dissipative source (the magnons) may interfere as an analog to the case of optical control and the electromagnetic vacuum acting on a confined two-level system.[53,54] The following process may be considered an exact solution of the macro-spin Hamiltonian, Eq. (1). It is also an illustration of the process described in Sec. 2.2.

The initial state, as shown in Eq. (2), is a product state of the current and the magnet. The time evolution of the macro-spin state may be expressed in terms of a set of Kraus operators,[55] namely reduction of the time evolution operator of the joint quantum system of the current electrons and the magnet to a set of operators acting on the macrospin conditioned on the current electron states. This is used to derive the Fokker-Planck equation of a probability distribution function of the magnetization orientation, $\hat{\mathbf{m}}$ in the direction of the macrospin (Θ, Φ). We use for our purpose, the P-representation,[56] $\mathcal{P}_J$, out of three possibilities included the P, Q, or Wigner representation. Then, the Fokker-Planck equation is of the form,

$$\frac{\partial}{\partial t}\mathcal{P}_J(\hat{\mathbf{m}}, t) = -\nabla \cdot (\mathbf{T}\mathcal{P}_J) + \nabla^2(\mathcal{D}\mathcal{P}_J), \tag{3}$$

where the drift vector,

$$\mathbf{T} = \mathcal{A}(\hat{\mathbf{m}} \times \mathbf{S}) \times \hat{\mathbf{m}} + \mathcal{B}\hat{\mathbf{m}} \times \mathbf{S}, \tag{4}$$

contains the two well-known terms of STT, $\mathbf{S}$ is the Bloch vector, i.e., the normalized vector expectation values of the current electron spin, the diffusion coefficient

$$\mathcal{D} = \mathcal{A}(1 - \hat{\mathbf{m}} \cdot \mathbf{S})/(2J + 1), \tag{5}$$

comes from the quantum fluctuation generated by the scattering[13] in Sec. 3.1.1. The parameters $\mathcal{A}$ and $\mathcal{B}$ are parameters from the Kraus operators. The diffusion term contains a factor $1 - \hat{\mathbf{J}} \cdot \mathbf{S}$ leading to the fluctuation and dissipation effects depending on the angle between the current polarization and the magnetization. The effects are maximal when the magnetization and the current spin are antiparallel and minimal when they are parallel. This gives the quantum explanation for the magnet configuration dependence of the shot noise in Sec. 3.1.1.

In the limit of large total spin J, the drift term is of the order $O(1/J)$ and the diffusion term of the order $O(1/J^2)$. In the equation, these parameters are compensated by the elongation of time scale and length scale. The equation becomes the Hamilton-Jacobi equation with the diffusion term also represent. Without the diffusion term, the semiclassical equation results.

3.2 A Model of Macro-spins and Magnons

There are at least two important recent semiclassical predecessors to the quantum approach to adding excited states to the macro-spin dynamics. One[57] adds magnons to the semiclassical magnetization dynamics by expanding the magnons as small spin deviations from any fixed orientation and compensates with an effective Hamiltonian for the geometrical phase brought about by the rotation from a fixed frame. The other[58] uses the analog of the electric field operator in terms of the photon operators for the connection of the z component of the spin to a boson representation and constructs a restriction for the bosons to reproduce the spin operators' commutation rules. The author, Peng, was motivated by representation of the classical magnetization state by a coherent state. I believe that he meant this to be the Glauber state rather than the spin coherent state[45] discussed in Sec. 3. In the limit of large number of spins, the two are the same. Thus, one might view Peng's construction as using the classical limit of the magnetization dynamics as a starting point to construct a quantum model but in the small nanomagnet regime, it would be interesting to have further study to determine whether the magnon coherent state or the spin coherent coherent state is the more appropriate representation of the actual physical state.

By contrast, our approach is to start with the quantum Hamiltonian and to construct the excited states quantum mechanically, introducing magnons à la Bloch[46] and Dyson.[47] For small spin deviation from a fixed orientation, the leading orders agree with the Holstein-Primokoff approximation. The Hamiltonian is composed of three parts: the current electrons, the magnet spin Hamiltonian, and the interaction with the magnetic spins, respectively,

$$\hat{H} = \hat{H}_e + \hat{H}_m + \hat{H}_i, \quad (6)$$

$$\hat{H}_e = \int d^3r\, \hat{\psi}^\dagger(\mathbf{r}) \left\{ -\frac{1}{2}\nabla^2 \right\} \hat{\psi}(\mathbf{r}), \quad (7)$$

$$\hat{H}_m = \hat{H}_0 + \sum_{\mathbf{q}} \omega_{\mathbf{q}} \hat{b}^\dagger_{\mathbf{q}} \hat{b}_{\mathbf{q}}, \quad (8)$$

$$\hat{H}_i = \int d^3r\, \hat{\psi}^\dagger(\mathbf{r}) \left\{ \left[\lambda_0(\mathbf{r})\hat{s}_0\hat{\mathrm{J}}_0 + \lambda(\mathbf{r})\hat{\mathbf{s}}\cdot\hat{\mathbf{J}} \right] + \left[\hat{s}_-\hat{b}(\mathbf{r}) + \hat{s}_+\hat{b}^\dagger(\mathbf{r}) \right] + \left[\hat{s}_z\hat{b}^\dagger(\mathbf{r})\hat{b}(\mathbf{r}) \right] \right\} \hat{\psi}(\mathbf{r}). \quad (9)$$

The current electron part in Eq. (7) is approximated by a free electron model with $\hat{\psi}^\dagger(\mathbf{r})$ as the spinor creation operator, with two spin components. The magnet spin part $\hat{H}_m$ in Eq. (8) is split into a rigid macro-spin part $\hat{H}_0$ and the magnon part. The magnon part can be treated rigorously following Bloch/Dyson by the magnet spin lowering operator $\hat{J}_{-,\mathbf{q}}$ which is a Fourier transform of the local spins.[59] In Eq. (8), the magnon part is treated in the non-interacting boson approximation with the creation operator $\hat{b}^\dagger_{\mathbf{q}}$ used by Tay and Sham[60] for computation. The current electron-magnet interaction term in $\hat{H}_m$, Eq. (9), is in the same boson approximation[60] for small spin deviation from the instantaneous magnetization direction. The terms in the first square brackets are the interaction of the current electron with the macro-spin as in Eq. (1), the second brackets the mutual spin flips, and the last brackets the two magnon term which affects the longitudinal spin components of both control and magnet. To study the effects of driving and dissipation,[60] the coupling of spin flip in macro-spin states (which can be represented by the uniform combination of a single site spin flip $\hat{b}^\dagger_{\mathbf{q}=0}$) and the magnon caused by impurity, dipolar external field and anisotropy.

The rest of this subsection is devoted to the discussion of simulation results with the Hamiltonian described above.[60] The dynamics of the magnetization now involved not only the recoil due to the scattering but also energy transfers with the current. The same quantum Monte Carlo simulation was carried out as Ref. 13 but it included the magnons.[60] The magnetization reversal is affected by the magnons in two competing factors. The reversal rate is increased by the thermal reduction of the magnetization magnitude but decreased by the damping effect of magnon emission.

The magnet temperature defined by the total energy was calculated[60] as a function of driving time by the current. It showed rapid increase from room temperature by about 50% during the first half of the magnetization reversal and slowing rate of increase during the second half. If other sources of interaction, such as phonon-assisted spin damping, are included, the magnet temperature might drop in the second half of the reversal.

The damping rate Γ on the magnetization dynamics was calculated from the impurity interaction by the Weisskopf-Wigner approximation. The damping term would appear to be the same as in the Landau-Liftshitz equation except that, in the microscopic case when the Markovian assumption is removed from the derivation, the damping constant goes to zero with the precession frequency.

A simulation study of the effects of Gilbert damping versus the impurity damping Γ was carried out for the normal current on a magnetic thin film with each-plane anisotropy. No external field nor other anisotropy were included. The STT driven precession increases linearly with the current but the self-consistent treatment of the Γ damping shows a critical threshold for precession with a steep initial rise in frequency and then decrease of the rising rate.

The final quantum Monte Carlo simulation for our discussion is on a magnetic thin film with a somewhat stronger in-plane anisotropy than perpendicular to the film under an external magnetic field in-plane and at a small angle to the in-plane easy axis. The Γ damping was included. The frequency dependence of the STT-driven precession on current density has a critical current value. Below the critical current, the precession is in a regular circle with a nearly constant frequency. Above the critical current, the frequency decreases with increasing current and the precession orbit is highly distorted. The dynamics is similar to the experiment[61] whose feature can also be found by semiclassical simulation. The fluctuation in the quantum approach has yet to be studied.

4. CONCLUSION

I have attempted to present a clear quantum picture of the rigidly rotating states of magnetization, their relation with the excited states of the magnet which present a combination of reduced polarization and dissipation, and the relation of all the magnet states with the control system. Although the control is restricted to current, the principles should apply to optical control. Measurements are expected to be similar in essential aspects to control. So far, our numerical simulation effort has demonstrated the possibility of quantum simulation of magnetization mean dynamics, fluctuations and correlation with the driving system. The fluctuations and correlation between the driving and the driven parts probably provide the essential differences between quantum and semiclassical theory but a smoking gun has not yet been found for the quantum theory. It is early days yet on numerical simulation of the quantum magnetization dynamics. Probable avenues lie in:

1. the quantum magnetic states, such as ferromgnets with hybridized s and d electrons, semiconductor ferromagnets, antiferromagnets in general, the Bose-Einstein condensation of magnons;
2. correlated states between magnet and control or measurement, such as the back action;
3. fluctuation and noise;
4. measurements of different components of the interacting control and magnetic systems;
5. study and design of smaller and smaller magnetic dots;
6. faster and faster operations and readouts.

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