

Research and design of the angle sensor based on optical arm amplification

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ABSTRACTS

Angle measurement is widely used in various fields as an important part of geometric metrology. Optical angle measurement methods stands out from many angle measurement methods because of its high measurement accuracy. Due to the limitation of grating line density, it is difficult to further improve the measurement accuracy of grating angle measurement method which was widely used in industrial field. The creatively new angle measurement method combined the regular pyramid with the right-angle reflector is proposed in this paper. The rotation angle of the measured object is transformed into a rotating laser through the regular pyramid, the variation of the rotating laser is geometrically amplified through the right angle mirror to realize the amplification of the variation of the angle and laser displacement is measured by position sensitive detector (PSD). The measured angle is divided into several equal parts by the pyramid, which reduces the angle measurement range of each PSD. Through the alternating measurement of several PSDs, the large range of continuous angle measurement can be achieved. The measurement principle of angle sensor based on optical arm amplification is researched in this paper, the mathematical model is established, and the correctness of the principle is verified through simulation analysis and actual measurement experiment by the prototype of angle sensor. The experimental results showed that the measurement error is ± 2 "in the range of 0.78° .

Keywords: angle measurement, regular pyramid, right angle mirror, PSD

1. INTRODUCTION

Angle measurement has always been an indispensable part of mechanical manufacturing and precision engineering. The accuracy of measurement results has a direct impact on the quality of products. In order to obtain more accurate measurement results, various angle measurement methods emerge one after another. Filatov Y V [1] presented the results of calibration of optical encodes of various designs, a laser dynamic goniometer (LDG) based on the ring laser gyroscope were designed and discussed. The results showed that the random error of the angle measurement does not exceed $0.3''$. Because of the complex structure of ring laser and costly price, its application filed was limited. According to incomplete statistics, according to different measurement principles, angle measurement techniques can be roughly divided into 16 categories, which can be divided further into mechanical angle measurement, electromagnetic angle measurement and optical angle measurement [2,3].

Although the optical angle measuring method appear later, it is more and more favored by researchers because of its high measurement accuracy. Grating angle measurement is widely used as a high-precision optical angle measurement method, but the measurement accuracy of grating angle measurement depends heavily on grating line density [4,5]. The higher line density can produce the higher measurement accuracy [6,7], but the higher manufacturing cost will be produced. Furthermore, there are eccentricity installation error and inclination error of circular grating, which need to be compensated. A method based on compressed sensing and sparse decomposition is proposed by Chen G [8], The experimental results validate that the proposed method can effectively improve the angle measurement accuracy of circular grating. Through the simulation and research of the eccentric error model and the tilt error model, the error compensation function is obtained, which reduces the peak value of the angle measurement error of a single reading head by about 93.76%, and the uncertainty was controlled to $2.08''$ [9].

In related research, Zhu Junchao [10] puts forward an angle measurement system based on the principle of laser self-mixing interference. This system combines the interference fringe counting method with the principle of laser triangulation,

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and the measurement accuracy is 11.16" within the range of $\pm 0.4^\circ$. Liu Yongkun [11] puts forward a reflective circular diffraction grating measuring system, which has a measuring accuracy of 10" within measuring range of $\pm 10^\circ$. Li Bo [12] puts forward a photoelectric reflection angle measuring system based on PSD, which can measure two-dimensional dynamic angle, the system can realize the measurement of $\pm 10^\circ$ swing angle range, the maximum system error is 25.2". The MLM-SMI angle measurement system proposed by Zhao Y, it has a resolution better than 28.8" in the range of $\pm 22^\circ$ [13]. In order to solve the contradiction between the radial size of the angle sensor and the measurement accuracy, a stereo grating angle sensor is proposed in reference [14], and the angle measurement error is 4.33 ".

A new angle measurement method is researched in this paper, which combines a regular pyramid with a right-angle reflector, and a PSD was used to achieve precise angle measurement. The new angle measurement method has high precision, simple structure and lower cost, its measurement precision could further improve by increase the number of right-angle reflector.

2. MEASURING PRINCIPLE

2.1 Optical arm amplification angle sensor

The angle between the bottom surface of the regular pyramid and the edge surface is 45° . By using the characteristics of the regular pyramid, laser incident to the reflecting surface of regular pyramid at 45° , and the reflected laser is perpendicular to the incident angle. For example, the laser is set above the regular pyramid to make the laser incident on a reflecting surface of the regular pyramid at 45° , the laser is reflected in the horizontal direction. The PSD is set on the laser reflection path, which can be used to measure the rotated angle of regular pyramid. When the regular pyramid rotates coaxially with the laser source, the position of the laser beam on the PSD changes accordingly. The rotated angle can be calculated by the position change of the laser beam on the PSD. The measurement principle is shown in Figure 1. In the figure, the solid line represents the position of a reflecting surface of regular pyramid before rotation and the red solid line represents laser reflection path. The dotted line represents the position of a reflecting surface of regular pyramid after rotation, and the dotted red line represents laser reflection path, the symbol L represents the distance between the PSD and the rotation center, and the symbol m represents the change of the position of the laser beam on the position detector before and after rotation, the symbol α represents the coaxially rotation angle by the regular pyramid and the laser source. Through L and α , the angle formula that the regular pyramid rotates coaxially with the laser source can be obtained, as shown in formula (1). Because of the length limitation of PSD, the angle measurement range of a PSD is limited. By using the geometric characteristics of pyramid, the measured angle is divided into several equal parts by the pyramid, which reduces the angle measurement range of each PSD. Through the alternating measurement of several PSDs, the large range of continuous angle measurement can be achieved.

$$\alpha = \arctan\left(\frac{m}{L}\right) \quad (1)$$

2.2 The displacement amplification principle of right-angle reflector

The PSD is widely used in the field of displacement measurement. The relative displacement of the measured object is indirectly obtained by measuring the position of the incident laser spot by PSD. At present, the displacement accuracy of PSD is difficult to break through $1\mu\text{m}$ due to the limitations of manufacturing technology. So a new displacement amplification principle combined with right-angle reflector was proposed by the author. The displacement can be amplified by a right-angle reflector, and its principle is shown in Figure 2.

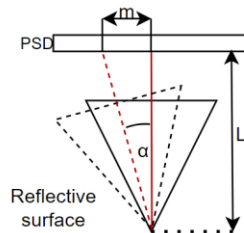


Figure. 1. Measurement principle of optical arm amplification angle sensor.

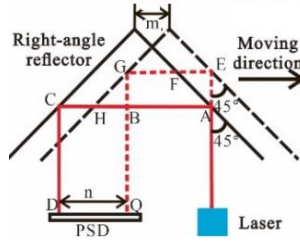


Figure 2. Schematic diagram of displacement amplification of right-angle reflector.

The laser is incident on a right-angle surface of a right-angle reflector the 45°, and after twice reflections, it is incident on the photosensitive surface of a PSD. When the right-angle reflector moves distance m , the m is a variable as distance change before and after movement of the right-angle reflector, the laser spot moves distance n on the photosensitive surface of PSD, the n is a variable as distance change before and after movement of laser spot. Formulas (2) can be obtained from the geometric relationship of Figure 2.

$$\begin{cases} m = EF = EA = BG = BH = CH \\ n = BH + CH = 2m \end{cases} \quad (2)$$

After the light beam passes through the right-angle reflector, the right-angle reflector is moved m distance in the direction perpendicular to the light beam, and the light spot is moved $2 \times m$ distance in the same direction. Thus the double displacement magnification is realized. Similarly, based on the same principle, the displacement can be further amplified by adding the number of right-angle reflectors. The quadruple displacement amplification principle with right-angle reflector is shown in Figure 3.

The laser is incident on a right-angle surface of the right-angle reflector 1 at an 45°, reflected twice in the right-angle reflector 1, then the laser is incident on a right-angle reflection surface of the right-angle reflector 2, reflected twice in the right-angle reflector 2, and it is incident again in the right-angle reflector 1, finally incident on the PSD photosensitive surface after being reflected twice in the right-angle reflector 1. At the same time, when the right-angle reflector 1 is moved distance m , the laser spot is moved distance n on the photosensitive surface of PSD. Equations (3) can be obtained from the geometric relationship.

$$\begin{cases} m = AE = FG \\ BF = DG = 2m \\ BD = AC = 5 \\ n = CE = AC - AE = 4m \end{cases} \quad (3)$$

It can be obtained from the formula (3) that when the right-angle reflector 1 moves distance m , the light spot moves distance $4 \times m$ on PSD, so the displacement is magnified four times.

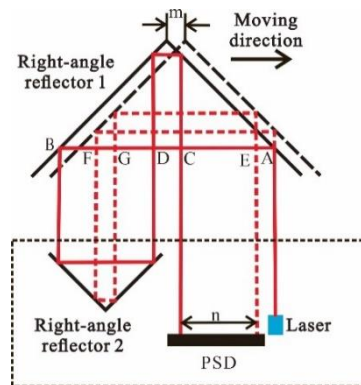


Figure 3. Diagram of quadruple magnification of right-angle reflector displacement.

2.3 Measurement principle of angle sensor based on optical arm amplification

The displacement amplification principle of right-angle reflector was applied to the angle sensor. For the convenience of principle description, the regular pyramid is placed horizontally, and a laser beam is incident on the reflector surface of the regular pyramid at an 45°. Because the angle between the regular pyramid and the reflector surface is 45°, the reflected

beam will be reflected into the horizontal plane. The reflected light beam is incident on the right-angle reflector, and after twice reflections in the right-angle reflector, it is incident on the PSD. The structure of angle sensor is shown in Figure 4. The attitude of angle sensor can change by angle measurement requirements.

When the regular pyramid and the laser source rotate around the central axis of the regular pyramid, the position of the light spot on the PSD will change accordingly. The angle change of the regular pyramid can be obtained by the displacement change of the light spot on the PSD. The principle diagram of angle measurement based on optical arm amplification is shown in Figure 5.

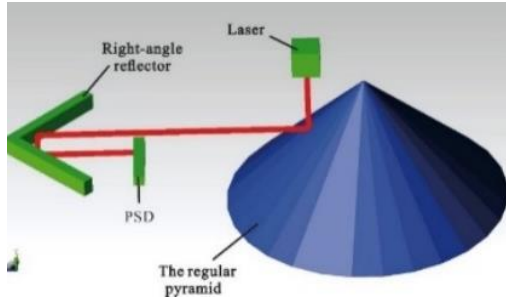


Figure 4. Structure of the angle sensor.

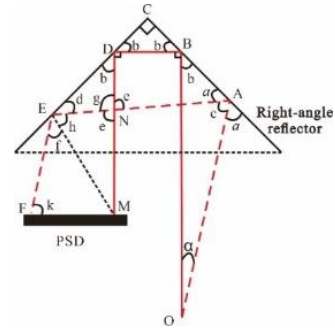


Figure 5. Schematic diagram of angle measurement.

At the initial position, the incident point of the laser beam to the right-angle reflector is point B, and after the regular pyramid rotates with the laser, the incident point of the laser beam to the right-angle reflector is moved to point A. The incident points of the rotating laser beam on PSD are F point and M point respectively. Assume that OB, DM and CB are known in the figure 4, and $OB=m_1$, $CB=CD=m$, $DM=m_2$, and $b = 45^\circ$ (in the figure 4, the angle is indicated by lowercase letters, and the intersection point is indicated by uppercase letters).

In $\triangle OBA$, $\triangle ACE$, $\triangle END$, formula (4) to (6) can be obtained from sine theorem.

$$\begin{cases} BA = \frac{OB \times \sin(\alpha)}{\sin(a+c)} = \frac{OB \times \sin(\alpha)}{\sin(135^\circ - \alpha)} \\ CA = CB + BA \end{cases} \quad (4)$$

$$\begin{cases} CE = \frac{CA \times \sin(\alpha)}{\sin(d)} = \frac{CA \times \sin(45^\circ + \alpha)}{\sin(45^\circ - \alpha)} \\ DE = CE - CD \end{cases} \quad (5)$$

$$\begin{cases} EN = \frac{DE \times \sin(b)}{\sin(g)} = \frac{DE \times \sin(45^\circ)}{\cos(\alpha)} \\ DN = \frac{DE \times \sin(d)}{\sin(g)} = \frac{DE \times \sin(45^\circ - \alpha)}{\cos(\alpha)} \end{cases} \quad (6)$$

The formula (7) can be obtained.

$$NM = m_2 - D \quad (7)$$

At this time, connecting EM, in $\triangle ENM$, formula (8) to (9) can be obtained by cosine theorem or sine theorem.

$$\begin{aligned} EM &= \sqrt{EN^2 + NM^2 - 2 \times EN \times NM \times \cos(e)} \\ &= \sqrt{EN^2 + NM^2 - 2 \times EN \times NM \times \sin(\alpha)} \end{aligned} \quad (8)$$

$$h = \arcsin\left(\frac{NM \times \sin(e)}{EM}\right) = \arcsin\left(\frac{NM \times \cos(\alpha)}{EM}\right) \quad (9)$$

The formulas (10) can be obtained.

$$\begin{cases} f = 180^\circ - 2d - h = 90^\circ + 2\alpha - h \\ k = 360^\circ - 90^\circ - f - h - e = 90^\circ - \alpha \end{cases} \quad (10)$$

In $\triangle EFM$, formula (11) can be obtained from sine theorem.

$$FM = \frac{EM \times \sin(f)}{\sin(k)} = \frac{EM \times \cos(2\alpha - h)}{\cos(\alpha)} \quad (11)$$

There will be position errors between the right-angle reflector and PSD. Assume that the angle between the optical path of the laser beam incident on the right-angle reflector and the ideal optical path is β_1 , and the angle between the PSD position and the ideal position is β_2 . The schematic diagram is shown in Figure 6.

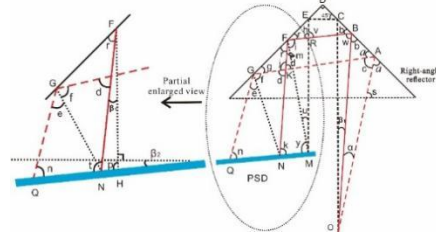


Figure 6. Measurement schematic diagram with position errors.

At the initial position, the incident point of the laser beam to the right-angle reflector is point B, and after the laser rotates with the regular pyramid, the incident point of the laser beam to the right-angle reflector is changed by point A. The incident points of the rotating laser beam on PSD are N point and Q point respectively. Assume that OC, EM, DC are known in the Figure 5, and OC=m1, EM=m2, DC=DE=m (in the figure 5, the angle is indicated by lowercase letters, and the intersection point is indicated by uppercase letters).

The Formula (12) and Formula (13) can be obtained from sine theorem.

$$\begin{cases} FK = \frac{FG \times \sin(g)}{\sin(j)} = \frac{FG \times \sin(45^\circ - \alpha - \beta_1)}{\cos(\alpha + 2\beta_1)} \\ GK = \frac{FG \times \sin(r)}{\sin(j)} = \frac{FG \times \sin(45^\circ - \beta_1)}{\cos(\alpha + 2\beta_1)} \end{cases} \quad (12)$$

$$i = \arcsin\left(\frac{EM \times \sin(45^\circ)}{FM}\right) - r = \arcsin\left(\frac{EM \times \sin(45^\circ)}{FM}\right) - (45^\circ - \beta_1) \quad (13)$$

It can be obtained from the figure that the value of $i + r$ is greater than 90° , so the value of i is shown in Formula (14).

$$i = 180^\circ - \arcsin\left(\frac{EM \times \sin(45^\circ)}{FM}\right) - r = 135^\circ - \arcsin\left(\frac{EM \times \sin(45^\circ)}{FM}\right) + \beta_1 \quad (14)$$

In $\triangle FNM$, $\triangle GKN$, $\triangle GKN$, formula (15) to (17) can be obtained from sine theorem or cosine theorem.

$$FN = \frac{FM \times \sin(y)}{\sin(k)} = \frac{FM \times \sin(\beta_2 + i - \beta_1)}{\cos(\beta_1 + \beta_2)} \quad (15)$$

$$\begin{aligned} GN &= \sqrt{KG^2 + KN^2 - 2 \times KG \times KN \times \cos(d)} \\ &= \sqrt{KG^2 + KN^2 - 2 \times KG \times KN \times \sin(\alpha + 2\beta_1)} \end{aligned} \quad (16)$$

$$f = \arcsin\left(\frac{KN \times \sin(d)}{GN}\right) = \arcsin\left(\frac{KN \times \cos(\alpha + 2\beta_1)}{GN}\right) \quad (17)$$

In the process of angle rotation, the value of f will change around 90° , so we should pay attention to the value of f with the change of rotation angle.

In $\triangle GNQ$, formula (18) can be obtained from sine theorem.

$$QN = \frac{GN \times \sin(e)}{\sin(n)} = \frac{GN \times \cos(2\alpha + 2\beta_1 - f)}{\cos(\alpha + \beta_1 + \beta_2)} \quad (18)$$

According to the above formula, the angle can be calculated by displacement variation of PSD.

3. SIMULATION ANALYSIS

In order to more intuitively express the relationship between rotation angle and spot displacement on PSD, and the influence of angles β_1 and β_2 on the displacement of the spot on the PSD, the above process is simulated by the MATLAB. Set $m_1=20\text{cm}$, $m_2=10\text{cm}$ and $m=5\text{mm}$. In the ideal condition ($\beta_1 = \beta_2 = 0$) and the actual condition ($\beta_1 > 0$)

or $\beta_2 > 0$), when the regular pyramid rotates 1° , 2° and 3° respectively, the displacement of the spot on PSD is shown in Table 1.

Table 1. Displacement of light spot on PSD.

Rotation angle ($^\circ$)	Displacement of facula (mm)	Condition
1	5.3599	$\beta_1 = \beta_2 = 0^\circ$
2	10.7232	$\beta_1 = \beta_2 = 0^\circ$
3	16.0929	$\beta_1 = \beta_2 = 0^\circ$
1	5.3624	$\beta_1 = 0^\circ, \beta_2 = 1^\circ$
2	10.7313	$\beta_1 = 0^\circ, \beta_2 = 1^\circ$
3	16.1101	$\beta_1 = 0^\circ, \beta_2 = 1^\circ$
1	5.3886	$\beta_1 = 0^\circ, \beta_2 = 5^\circ$
2	10.7971	$\beta_1 = 0^\circ, \beta_2 = 5^\circ$
3	16.2288	$\beta_1 = 0^\circ, \beta_2 = 5^\circ$
1	5.3632	$\beta_1 = 1^\circ, \beta_2 = 0^\circ$
2	10.7330	$\beta_1 = 1^\circ, \beta_2 = 0^\circ$
3	16.1126	$\beta_1 = 1^\circ, \beta_2 = 0^\circ$
1	5.4092	$\beta_1 = 5^\circ, \beta_2 = 0^\circ$
2	10.8383	$\beta_1 = 5^\circ, \beta_2 = 0^\circ$
3	16.2908	$\beta_1 = 5^\circ, \beta_2 = 0^\circ$

From Table 1, there are a certain influence on the displacement of the light spot on the PSD from the installation errors of the angles β_1 and β_2 , and appropriately increasing angle of installation is conducive to expanding the displacement of the light spot on the PSD, so as to improve the angle measurement accuracy.

4. DESIGN OF ANGLE MEASUREMENT EXPERIMENTAL PLATFORM BASED ON OPTICAL ARM AMPLIFICATION

In order to verify the angle measurement principle based on optical arm amplification, an angle measurement experimental platform was designed in this paper. A semiconductor laser with wavelength of 650 nm and power of 5 mW was selected as light source. The hamamatsu S3932 model PSD was selected as measurement device, with a photosensitive area of 1×12 mm, a position resolution of $0.3 \mu\text{m}$ and a spectral response range of 320-1100 nm. The AD7606 analog-to-digital converter chip produced by ADI was used for data acquisition. Because of the high processing cost, long cycle and great difficulty of the regular pyramid, the trapezoidal mirror is fixed on the metal regular pyramid surface in this paper, so as to realize the purpose of the reflection of laser. The trapezoidal mirror is a front mirror without distortion and ghosting, which has good reflection effect. The circumscribed circle radius of the bottom surface of the regular pyramid with 28 edges has a of 25 mm, the height of regular pyramid is 24 mm. The parameters of trapezoidal reflector are an upper bottom of 3.2 mm, a lower bottom of 5.6 mm, a height of 15.1 mm and a thickness of 1 mm. The right-angle reflecting surfaces of the right-angle reflector are 30 mm length and 30 mm width.

The instruments XL-80 laser interferometer and XR20-W angle measurement component produced by British Renishaw Company were used to detect and record the rotation angle of the regular pyramid in the experiment, and the angle measurement accuracy was $\pm 1''$. A three-layer structure is designed, in which the angle measurement component is set on the top layer, the laser source is set in the middle layer, and the regular pyramid reflector is set at the bottom layer. Finally, the circular rotating platform drives the regular pyramid, the laser source and the angle measuring component to rotate coaxially. The physical picture of the experiment is shown in Figure 7.

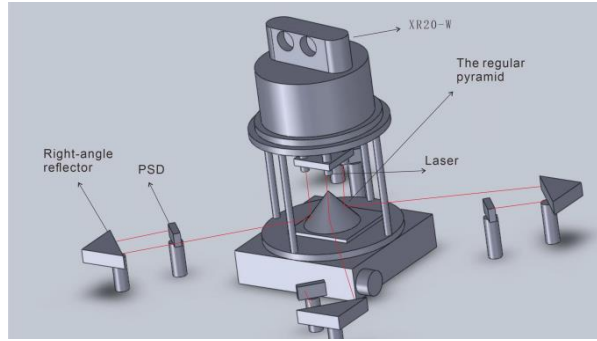


Figure 7. Experimental platform of optical arm amplification angle measurement.

5. EXPERIMENT AND DATA PROCESSING

Because the mathematical model is complex, especially there are unavoidable installation errors, the principle is verified by comparative experiment method. In the comparative experiment process, the rotating platform is driven to rotate once per 0.0025° , and after each rotation, the rotating angle value from laser interferometer and the position value (voltage ratio of PSD) of light spot on PSD are recorded, and a piecewise linear table of the position value and rotating angle value is established. Then, in the measurement process, the rotation angle of the regular pyramid is obtained by piecewise linear interpolation table according to the position value of PSD [9,10].

The angle values recorded by laser interferometer are taken as theoretical values, and the angle values calculated by the piecewise linear interpolation table are taken as measured values. During the experiment, the turntable is driven to rotate once per 0.03° , and the experiment is repeated four times by using different PSD, with the measurement range of 0.78° respectively. The measurement range is limited by the length of PSD and the installation position of PSD, when the PSD length increases or the PSD distance from the pyramid decreases, the measurement range can increase. The measured values are compared with the theoretical values, and the measured error values are shown in Figure 8. The experiment results show that the max measurement error is $\pm 2''$ in the range of 0.78° .

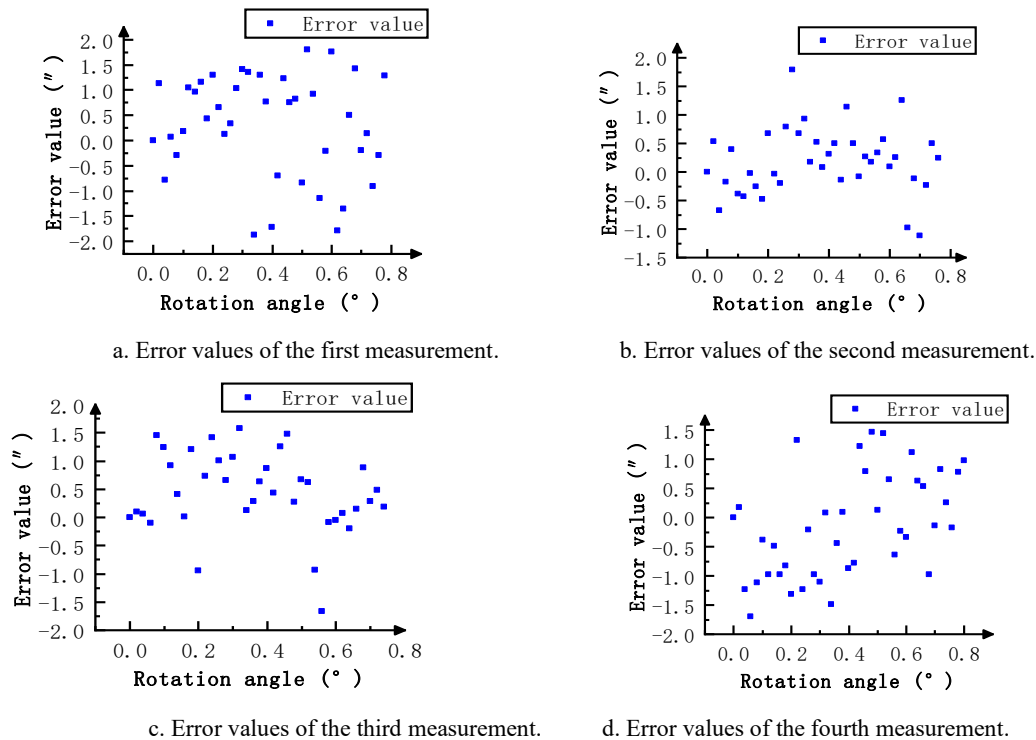


Figure 8. Measurement error values of angle measurement experiment.

The nominal error is a measurement accuracy index of the whole instrument, and the formula is shown in formula (23).

$$\delta_{max} = \frac{\max\{|\alpha_i - \theta_i|\}}{Y_{FS}} \times 100\% \quad (23)$$

Where: α_i is represents the measured value, and θ_i represents the theoretical value.

The nominal error of four measurements can be obtained from equations (68), as shown in Table 2.

Table 2. Error results of four angle measurement experiments.

Measurement	Nominal error
first measurement	0.0677%
second measurement	0.0641%
third measurement	0.0638%
fourth measurement	0.0590%

6. CONCLUSION

The new angle measurement method is research by innovatively combining the regular pyramid with the right-angle reflector in this paper, and the mathematical models of rotation angle and spot displacement on PSD are established. The principle of the angle measurement method with optical arm amplification is verified by experiments. Experiments show that the nominal error of this method is 0.0677%, and the max measurement error is $\pm 2''$ in the range of 0.78° . In the experiment, the length of the optical arm and the length of PSD can be increased to improve the measurement accuracy. The optical arm amplification angle measurement method has no range limitation in principle, the large range continuous angle measurement can be realized by alternately setting multiple groups of lasers and PSD.

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