Quantum Decoherence and Qubit Devices

Howard E. Brandt
U.S. Army Research Laboratory
2800 Powder Mill Road, Adelphi, MD 20783
Phone: (301)394-4143
Email: hbrandt@lamp0.arl.army.mil

ABSTRACT

A plethora of qubit devices is under consideration for possible development ranging from quantum teleporters to quantum computers. The primary obstacle to the success of these efforts is the phenomenon of quantum decoherence, the rapid vanishing of the off-diagonal components of the reduced density matrix representing the computational degrees of freedom of the device. Following a review of the physics of quantum decoherence, several instructive examples are exposited. Decoherence issues associated with the various possible approaches to quantum computing are addressed. Also, possible generic methods are reviewed for surmounting the decoherence obstacle.

Keywords: quantum information processing, quantum computers, qubits, quantum decoherence, qubit devices, qubit persistence probability.

1 INTRODUCTION

Qubit devices currently under consideration for possible development include small-scale quantum information processors such as quantum key receivers, quantum games, quantum teleporters, and also large-scale quantum computers capable of efficiently factoring very large numbers and retrieving data from extremely large data bases [1,2]. Quantum coherence must be maintained in qubit devices, and this is a primary obstacle to developing quantum information processors, small and large. For an operational quantum computer to be built, a critical requirement must be satisfied: the quantum register must be maintained in a coherent superposition of a very large number of states, so that the computational outputs interfere to result in a very high probability that when the computer output is read, it will yield the correct answer. Quantum coherence must be maintained throughout the calculation. However, the couplings of the qubits to both their internal and external environments will inevitably result in quantum decoherence. Even weak interactions with the environment may result in significant departures from unitarity for the qubit subsystems, with associated dephasing and loss of coherence. This quantum decoherence is the same quantum mechanism that led to a profound understanding of the macroscopic vs. microscopic dichotomy, in which quantum superpositions are virtually unseen in the macroscopic world, while they are the rule in the microscopic world of atoms and elementary particles [3,4]. Various long standing quantum paradoxes, such as the Schrödinger cat paradox or the problem of Wigner's friend, have also been largely resolved in terms of quantum decoherence [3,5]. The interaction of a complex object with its internal and external environments usually results in extremely rapid vanishing of the off-diagonal components of its reduced density matrix (expressed in an appropriate basis), and to the corresponding phenomenon of quantum decoherence [3,6].

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(This includes practically all macroscopic objects except coherent light beams and superconducting states [3].) For mesoscopic and microscopic systems such as qubit devices, the same decoherence phenomenon presents a formidable obstacle to the maintenance of quantum coherence.

The present paper includes revised, expanded and updated versions of parts of Refs. [1,2,7,8]. We first review in Section 2 the physics of quantum decoherence in generic two-state systems including environmental interactions. We discuss decoherence in terms of the rapid decay of the off-diagonal components of the system's reduced density matrix, resulting from rapid orthogonalization of the entangled environmental states. In Section 3, we formulate the persistence probability for a qubit device as the probability of measuring its computational degrees of freedom in the unperturbed state, avoiding the decoherence arising from environmental interactions. A decoherence time can be obtained from the persistence probability. In Section 4, we apply the persistence probability formalism to generic qubit devices coupled to a thermal field environment. In Section 5, characteristics of a quantum register are reviewed, and in Section 6, following a review of ion-trap quantum computers, an intricate calculation is sketched of one possible mechanism of quantum decoherence in a trapped-ion quantum register. Sections 7-12 address various other approaches to quantum computer development, including ones based on optics, cavity QED, NMR, silicon-based nuclear spin and quantum dots, Josephson junctions and SQUIDs, and neutral atoms, respectively. Section 13 addresses quantum error correction. Section 14 contains a summary.

2 QUANTUM DECOHERENCE

Consider a two-state quantum system in the absence of environmental interactions. The state vector $|\psi\rangle$ for such a two-state system, first considered to be closed, lies in a two-dimensional Hilbert space, and is given by

$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle , \tag{1}$$

where $|0\rangle$ and $|1\rangle$ are kets representing the two states, here also serving as orthonormal basis vectors, and α_0 and α_1 are complex numbers. The corresponding density operator ρ is given by

$$\rho = |\psi\rangle \langle\psi| . \tag{2}$$

Substituting Eq. (1) in Eq. (2), one obtains for the density operator of this two-state system

$$\rho = |\alpha_0|^2 |0\rangle \langle 0| + \alpha_0\alpha_1^* |0\rangle \langle 1| + \alpha_0^*\alpha_1 |1\rangle \langle 0| + |\alpha_1|^2 |1\rangle \langle 1| . \tag{3}$$

The corresponding density matrix is

$$[\rho_{mn}] = [\langle m|\rho| n\rangle] = \begin{bmatrix} |\alpha_0|^2 & \alpha_0\alpha_1^* \\ \alpha_0^*\alpha_1 & |\alpha_1|^2 \end{bmatrix} . \tag{4}$$

The diagonal components are the populations, and the off-diagonal components are the coherences. The populations measure the probabilities that the system is in either state, and the coherences measure the amount of quantum interference between the states. The expectation value of any observable represented by an operator A for the two-state system is given by

$$\langle\psi| A |\psi\rangle = \mathrm{Tr}(\rho A) = \sum_{mn} \rho_{mn} A_{nm} , \tag{5}$$

and it is clear that, in general, the coherences are as important as the populations in determining the expectation values of observables.

Generally, a system is not closed. It does not exist in absolute isolation, and possible interactions with both its external and internal environments must be taken into account. If the two states of interest are part of an object containing other internal degrees of freedom, the latter constitutes the internal environment, and the

external environment is external to the object. For complex systems, including most macroscopic and many mesoscopic systems, the two states of interest might themselves represent two collective observables [3]. Consider now, therefore, a two-state system with state vector $|\psi(t)\rangle$ at time t, including environmental interactions:

$$|\psi(t)\rangle = \alpha_0 |0\rangle \otimes |e_0\rangle + \alpha_1 |1\rangle \otimes |e_1\rangle , \tag{6}$$

in which now the two possible states of the system, $|0\rangle$ and $|1\rangle$, through unitary evolution, have become entangled with the correlated normalized environmental states $|e_0\rangle$ and $|e_1\rangle$, respectively. (Here $\otimes$ denotes the tensor product.) Thus, the density matrix representing the complete system with its environment, becomes

$$\begin{aligned} \rho(t) = {} & |\alpha_0|^2 |0\rangle \otimes |e_0\rangle \langle 0| \otimes \langle e_0| + \alpha_0 \alpha_1^* |0\rangle \otimes |e_0\rangle \langle 1| \otimes \langle e_1| \\ & + \alpha_0^* \alpha_1 |1\rangle \otimes |e_1\rangle \langle 0| \otimes \langle e_0| + |\alpha_1|^2 |1\rangle \otimes |e_1\rangle \langle 1| \otimes \langle e_1| . \end{aligned} \tag{7}$$

(Here and in the following, the term "density matrix" is used interchangeably with "density operator.") If one is interested only in what the two-state system is doing, and not the environment, one need only know the reduced density matrix of the two-state system, with the environmental states traced out. For this purpose, choose as environmental basis vectors the correlated vector $|e_0\rangle$ and also $|e_0^\perp\rangle$, orthogonal to $|e_0\rangle$, namely,

$$\langle e_0^\perp | e_0\rangle = 0, \quad \langle e_0 | e_1\rangle \equiv \cos\,\theta, \quad \langle e_0^\perp | e_1\rangle = \sin\,\theta, \quad \langle e_0 | e_0\rangle = 1, \quad \langle e_1 | e_1\rangle = 1. \tag{8}$$

The reduced density matrix $\rho_s(t)$ of our two-state system is then given by

$$\rho_s(t) = \mathrm{Tr}_e \rho(t) = \langle e_0 | \rho(t) | e_0\rangle + \langle e_0^\perp | \rho(t) | e_0^\perp\rangle , \tag{9}$$

where Tr_e denotes the trace over the environmental basis states. Substituting Eq. (7) in Eq. (9), and using Eqs. (8), one obtains

$$\begin{aligned} \rho_s(t) = {} & |\alpha_0|^2 |0\rangle \langle 0| + \alpha_0 \alpha_1^* \cos\,\theta\ |0\rangle \langle 1| \\ & + \alpha_0^* \alpha_1 \cos\,\theta\ |1\rangle \langle 0| + |\alpha_1|^2 (\cos^2\theta + \sin^2\theta) |1\rangle \langle 1| . \end{aligned} \tag{10}$$

If one uses the trigonometric identity, $\cos^2\theta + \sin^2\theta = 1$, Eq. (10) becomes

$$\begin{aligned} \rho_s(t) = {} & |\alpha_0|^2 |0\rangle \langle 0| + \alpha_0 \alpha_1^* \cos\,\theta\ |0\rangle \langle 1| \\ & + \alpha_0^* \alpha_1 \cos\,\theta\ |1\rangle \langle 0| + |\alpha_1|^2 |1\rangle \langle 1| . \end{aligned} \tag{11}$$

Comparing Eq. (11) with Eq. (3), one can see that as a result of including environmental interactions, the coherences each contain an additional factor of $\cos\,\theta$, the overlap between the correlated environmental states (see Eqs. (8)). The system and its environment evolve, interacting incessantly, and because of decoherence, the overlap between the environmental states $|e_0\rangle$ and $|e_1\rangle$ can become negligible; one then has orthogonalization of the environmental basis states, namely,

$$\cos\,\theta \equiv \langle e_0 | e_1\rangle \longrightarrow 0, \tag{12}$$

and Eq. (11) becomes

$$\rho_s(t) \underset{\cos\,\theta \longrightarrow 0}{\longrightarrow} |\alpha_0|^2 |0\rangle \langle 0| + |\alpha_1|^2 |1\rangle \langle 1| . \tag{13}$$

This is the phenomenon of quantum decoherence, resulting in orthogonalization of the correlated states of the environment and the vanishing of the off-diagonal components of the reduced density matrix [3,6]. For a complex macroscopic or mesoscopic two-state system, the orthogonalization Eq. (12) generally occurs extremely rapidly. For example, if Eq. (6) were to represent a macroscopic Schrödinger cat state in which $|0\rangle$ and $|1\rangle$ represent live and dead cat states, respectively, the reduced density matrix Eq. (11) would decohere to the diagonal form Eq. (13) so quickly that the coherences would never be observable. The reduced density matrix Eq. (13), as a result of the decoherence, becomes effectively a statistical mixture, and the paradoxical features of the Schrödinger cat paradox largely evaporate. "For all practical purposes," there is simply a probability $|\alpha_0|^2$ that the cat is alive, and a probability $|\alpha_1|^2$ that it is dead, with no mysterious interference between the live and dead cat states. Ontological uncertainty is effectively reduced to the ordinary epistemic uncertainty which we experience in everyday life.

The dynamical evolution represented by Eq. (13) is, of course, nonunitary because, although the evolution of the total density matrix representing a system and its environment evolves unitarily in accordance with the

Schrödinger equation, in general, a reduced density matrix does not. Of course, the details of the evolution (Eq. (13)) depend on the specific structure of the total Hamiltonian of the system together with its environment, including all possible interactions. For a macroscopic system, and also many mesoscopic systems having many degrees of freedom, the environment commonly has an enormous Hilbert space and a crowded energy spectrum. Heuristically, in terms of perturbation theory, close energy levels result in high sensitivity to perturbations. Two slightly different perturbations may lead to very different perturbed wave functions, which become orthogonal. Environmental wave functions have many variables, and vanishing wave-function overlap in one variable is sufficient for orthogonality. The Hilbert space of environmental states can become so enormous that two states are very unlikely to not be orthogonal. The resulting loss of phase correlations in the high-dimensional environmental configuration space results in orthogonalization of the environmental states that are correlated with the system states, and produces quantum decoherence.

As an example of a macroscopic two-state system interacting with its environment, it is instructive to consider an ordinary pendulum in a hypothetical superposition state, Eq. (6), for which the kets $|0\rangle$ and $|1\rangle$ represent, at the same time, the pendulum of mass M and period T_p with its lower end in two possible positions, separated by a distance D in an ambient environment with negligible temperature. The two states represent two collective observables, namely two positions of the center of mass of the pendulum. A dominant contribution to the environmental interaction is, in this case, internal, and results from excitation of vibrational modes in the pendulum instigated at the point of suspension. Other possible environmental contributions to decoherence include, for example, ambient radiation, and collisions with any surrounding molecules. Correlations set up with the environmental degrees of freedom destroy the coherence of the initial state, and result in rapid collapse into a localized state. A reservoir-driven open-system model [9] can be adapted [3] to the pendulum decoherence problem, and leads to the following simple formula for the decoherence time $\tau_{\rm dec}$ (lifetime of the coherences):

$$(\tau_{\rm dec}/\tau_{damp}) = \hbar T_p/(\pi M D^2), \tag{14}$$

where τ_{damp} is the classical pendulum damping time, and $\hbar$ is Planck's constant divided by 2π. For $M = 1$ gram, $D = 1$ cm, and $T_p = 1$ s, the decoherence time in Eq. (14) becomes $\sim 10^{-28}$ in units of the damping time. (Eq. (14) corrects the corresponding expression given on p. 291 of Ref. 3.) Thus, even if one academically considers a two-state system consisting of such a superposition of spatially separated pendulum states, it would decohere, for all practical purposes, instantaneously, and the superposition would thereby be effectively unobservable. The pendulum would simply be with some probability in one or the other position, with no mysterious interference and associated ontological uncertainty.

Since decoherence occurs so quickly in most macroscopic systems, it is typically far too difficult to observe; however, experimental evidence for the effect was indicated in early experiments with superconducting quantum interference devices (SQUIDS) [10], and more recently, it was definitely observed in a quantum optics experiment [11]. In the latter experiment, a mesoscopic superposition of quantum states involving coherent electromagnetic fields with classically distinct phases was produced, and its subsequent decoherence was observed.

Also for mesoscopic or even microscopic systems, decoherence often results in rapid vanishing of the off-diagonal components of the reduced density matrix for the system. This same decoherence phenomenon presents a formidable obstacle to the development of operational qubit devices such as quantum computers. The decoherence results from interactions between the qubits and the noncomputational degrees of freedom in both the internal and external environment. In the case of qubit devices based on atomic states, even spontaneous emission can be understood in terms of decoherence arising from interactions between the atom and the environment of vacuum fluctuations.

A number of methods are available for calculating decoherence in qubit devices, including the use of Lagrangian field theory, path integrals, master equations, quantum Langevin equations, short-time perturbation expansions, Monte-Carlo methods, semiclassical methods, exact density matrix solutions, and phenomenological methods.

In the Lagrangian field theory approach, a Lagrangian model is adopted to represent the qubit device interacting with its external and internal environments, which can be represented by fields. The evolution of the

environmental fields is obtained from the Heisenberg equations of motion. The corresponding total Hamiltonian is used to calculate the time evolution of the reduced density matrix in which the environmental fields are traced out. Any decaying time evolution of the coherences thereby follows. The Lagrangian field theory approach was used in an early model calculation of decoherence in a generic computer memory [12].

In the path integral approach to calculating decoherence, the time evolution of the qubits, together with their environment, is expressed as a Feynman path integral, from which one can construct the time evolution of the density matrix in the form of a double path integral [13-21]. Tracing over the environmental coordinates, one obtains the reduced density matrix for the qubits expressed as a double path integral with the Feynman-Vernon influence functional in the integrand. The path integral approach expressed in terms of the influence functional can, for example, be used to derive a master equation for an object coupled to a general environment [22]. In more recent work, the path integral approach has been used to calculate the coherences and populations of a driven damped two-state system [23]. The path integral approach has been used specifically to calculate decoherence in an ion trap quantum computer (See Sections 3 and 6).

In the master equation approach [4,6,24-33], one uses the fundamental quantum mechanical equation for the density matrix (expressed in terms of the total Hamiltonian describing the qubit interaction with its environment) to obtain an equation for the reduced density matrix (expressed in terms of qubit variables) by tracing out the environmental variables. One can often approximate the resulting integrodifferential equation by truncating an iterative solution involving various orders in the interaction. A Markov approximation is also often made, which reduces the integrodifferential equation to a local differential equation for the reduced density matrix representing the qubit. The resulting equation is used to calculate the time evolution of the coherences for the qubit device. The master equation approach has been used to calculate the evolution of coherences and populations for a system coupled to a heat reservoir with a Debye density of states [28].

In the quantum Langevin equation approach, the Hamiltonian of the system interacting with its environment (represented as a heat bath of oscillators) is used together with the Heisenberg equation of motion to obtain differential equations that may be integrated to produce decoherence times [16,27,30-32].

In the short-time perturbative expansion approach, the Hamiltonian for the total system of the qubits, together with their environment, is used to express the time evolution of the total density matrix in terms of its initial state. One obtains the reduced density matrix describing the qubits by tracing over the environmental degrees of freedom. The idempotency defect of the reduced density matrix can then be written as a short-time power series expansion, from which decoherence times can be obtained. Successive terms of the perturbative expansion generate times for decorrelation processes involving successive powers of the Hamiltonian. Second-order results can be shown to exactly reproduce expressions for decoherence times obtained by other more involved and indirect approaches [34].

In the Monte Carlo wave function method, the environment is treated stochastically and introduces random fluctuations in the qubit wave function. The density matrix is expanded in terms of the wave function, and a stochastic average is taken over the phase fluctuations within a given period of time. The time evolutions of the coherences and the populations follow. The Monte Carlo wave-function approach has been shown to be equivalent to the master-equation treatment [35]. In order to control numerical fluctuations, a large number of wave functions must be propagated, and the simulations may be more or less efficient than the master equation approach, depending on the dimensionality of the system Hilbert space and the type of observable calculated. Stochastic methods are very useful in calculating quantum evolutions, and generally lead to a description in terms of a stochastic wave-function evolution, which may either involve quantum jumps [35-39] or be continuous [40,41]. Several recent reviews thoroughly document quantum jump, Monte Carlo wave function, and quantum-trajectory methods [30,31,42,43]. The quantum jump description can also be formulated in the framework of stochastic differential equations using the Ito calculus [32,44].

In the semiclassical approach, the environment of the qubit device is treated approximately as a fluctuating classical field interacting with the device. An ensemble average is taken over all possible field histories appearing

in the density matrix. The effect of the averaging is to destroy the coherences. The semiclassical approach cannot, however, correctly model the effects of thermal and vacuum fluctuations, which demand a full quantum approach. An example of the use of the semiclassical approach is an estimate of qubit decoherence due to a reservoir consisting of a classical magnetic field [45].

In the exact density matrix approach, one obtains explicit density matrix solutions by representing the density matrix in terms of appropriate dynamical operators of the system without first taking the usual trace over the environment. Finally, one traces over the environment to obtain the qubit coherences. This approach has been used to examine the behavior of the unreduced density matrix of several simple open quantum systems, including the detailed behavior of a background thermal bath [46].

Before further discussing the physics of quantum decoherence in qubit devices, it is well to recall the characteristic requirements for a qubit in quantum information processing: (1) it is a two-state system, (2) it can exist in a superposition of Boolean states, and (3) it can be entangled with the states of other qubits [47]. To implement a useful qubit in a quantum computer, it is not enough that its embodiment be scalable, but it must robustly have all three characteristics. A qubit has the same general form as that discussed above in Eqs.(1) to (13), and is likewise subject to the phenomenon of quantum decoherence. A simple example of a qubit is that of two polarization states $|\uparrow\rangle$ and $|\rightarrow\rangle$ of a photon, in which the two kets correspond to vertical and horizontal polarizations, respectively. A general superposition state of the qubit is $|\psi\rangle = \alpha\,|\uparrow\rangle + \beta\,|\rightarrow\rangle$, corresponding to the Boolean state $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$. The entanglement of two such qubits can be represented by $|\psi\rangle = \alpha_{12}\,|\uparrow\rangle_1\,|\uparrow\rangle_2 + \beta_{12}\,|\uparrow\rangle_1\,|\rightarrow\rangle_2 + \gamma_{12}\,|\rightarrow\rangle_1\,|\uparrow\rangle_2 + \delta_{12}\,|\rightarrow\rangle_1\,|\rightarrow\rangle_2$, where the coefficients are some complex numbers, and the state $|\psi\rangle$ cannot be factored into a single product of states of the two qubits. Entanglement results from interactions represented by unitary transformation [1]. Some examples of possible qubit implementations include (1) ions in an electromagnetic trap (here a qubit may correspond to two hyperfine ground states of a laser-cooled trapped ion, or to two quantized-harmonic-oscillator vibrational states of a trapped ion (phonons), (2) cavity QED devices (here a qubit may correspond to two photon polarization states interacting with an atom in a low-loss QED cavity), (3) Ramsey atomic interferometric devices (here a qubit corresponds to two atomic states that can be coupled to another qubit consisting of two states of the quantized electromagnetic field in a superconducting high-Q microcavity), (4) dual single-photon paths (here a qubit corresponds to two possible paths of a single photon in an optical circuit), (5) devices based on nuclear or electron spins in an external magnetic field (here a qubit may correspond to two spin states of a particle in an external magnetic field), (6) quantum dot in a static magnetic or electric field (here a qubit corresponds to either two single-electron spin states, or two states of an electric dipole in the presence of the field), (7) Josephson junction devices (here a qubit corresponds to two charge states), and (8) superconducting quantum interference devices (SQUIDs) (here a qubit corresponds to two magnetic flux states linking a superconducting loop).

A trapped-ion qubit device provided the first experimental demonstration of a fundamental quantum logic gate, operating on prepared quantum states [48]. Mesoscopic "Schrödinger cat states" were also constructed by the use of a trapped ion [49]. Also, some time ago, a cavity-QED qubit device was employed to experimentally demonstrate the conditional dynamics required for a quantum phase gate involving two photonic qubits, both traversing a microcavity [50]. An approach using Ramsey atomic interferometry was also used to experimentally demonstrate conditional quantum dynamics at the single quantum level [51]. Schrödinger cat states have also been produced using Josephson junctions and SQUIDs [52,53], and also using large gaseous ensembles of atoms [54]. Quantum teleportation of qubit states has also been accomplished [55]. The entanglement of more than a few qubits remains to be demonstrated in most approaches. Some estimates of typical single-qubit decoherence times for various qubit implementations are as follows [56]: (1) trapped ions, 10^{-1} s, (2) electron spins, 10^{-3} s, (3) nuclear spins, 10^{4} s, and (4) electron quantum dots, 10^{-6} s. One should emphasize that such estimates are often very rough, and far too little reliable data are available on decoherence in mesoscopic and microscopic systems appropriate for qubit device implementations. Some possible physical mechanisms contributing to quantum decoherence in qubit devices include the following (without regard for redundancy or relative degree of specificity): interactions with internal and external environments, interactions with noncomputational degrees of freedom, spontaneous emission, interactions with electromagnetic fluctuations, laser-beam power fluctuations, interactions with ambient background radiation, interactions with vibrational excitations (phonons), mechanical

fluctuations, collisions with background atoms, interactions with contaminants, thermal fluctuations, and fluctuations in quantum tuning. Possible methods for decoherence amelioration include qubit isolation from the interacting background environment, quantum error correction and control methods, operating in decoherence-free subspaces, increased computer speed, and entanglement purification (See Section 13). Interactions between the computational degrees of freedom of a qubit device and environmental degrees of freedom result in damping of the off-diagonal components of the reduced density matrix in the computational basis, and in the phenomenon of decoherence. Qubit device designs that minimize or avoid such interactions may lead to increased decoherence times. In the quantum error-correcting code approach to decoherence amelioration, a set of qubits is encoded by being mapped unitarily into a subspace of a larger quantum state space, consisting of a larger set of qubits, so that if any of the encoded qubits suffer decoherence, the resulting set of qubit states in the larger space can be used to faithfully reconstruct the initial quantum state of the encoded qubits. By operating in decoherence-free subspaces of the system Hilbert space, the coupling of the qubits to decohering interactions can be significantly reduced. Also, since successful quantum computation requires that the compute time not be greater than the time coherence can be effectively maintained for the computational degrees of freedom of the device, it follows that increased computer speed may tolerate shorter decoherence times. In entanglement purification, entangled qubit states with degraded fidelity are distilled to a smaller number of high fidelity states by means of local unitary operations and measurements together with postselection. Although decoherence is widespread in macroscopic and mesoscopic systems, it does not have a universal description [57] (without special assumptions, as in Ref. [58], for example). It is therefore important that the phenomenon of quantum decoherence be much more extensively and thoroughly investigated, both experimentally and theoretically, if useful qubit devices are to become a reality. In the next section, the qubit persistence probability is formulated as the probability for the qubits in a qubit device to remain in their unperturbed state, avoiding the decoherence arising from environmental interactions (without quantum error correction).

3 QUBIT PERSISTENCE PROBABILITY

Consider a qubit device along with its environment, both internal and external. Denote the total Hamiltonian of the qubit device along with its environment by

$$H = H_q + H_e + H', \tag{15}$$

where H_q is the unperturbed qubit Hamiltonian expressed in terms of the computational degrees of freedom of the qubit device, H_e is the contribution of the environment to the total Hamiltonian, and H' is the interaction term. The environmental term H_e includes both the internal environment of noncomputational degrees of freedom of the qubit device, as well as those of the external environment. Denote the initial computational state of the qubit device (at time $t = 0$) by $|\text{in}\rangle$. We refer to this in the following as the initial qubit state. We define the qubit persistence probability $P(t)$ at time t as the probability of measuring, at time t, the final computational state $|\text{fin}\rangle$ of the qubit device to be in its unperturbed state without the decoherence arising from environmental interactions. Specifically, the unperturbed qubit state $|\text{fin}\rangle$ at time t is given by

$$|\text{fin}\rangle = e^{-\frac{i}{\hbar}H_q\ t}\,|\text{in}\rangle\,, \tag{16}$$

written in terms of the unperturbed qubit evolution operator in the Schrödinger picture. Also, we denote by $\rho_q(t)$ the reduced density matrix for the computational state of the qubit device at time t, with the environmental states traced out. The qubit persistence probability $P(t)$ is then defined by

$$P(t) = \langle\text{fin}|\,\rho_q(t)\,|\text{fin}\rangle\,. \tag{17}$$

The evolution of the reduced density matrix $\rho_q(t)$ can be obtained from the total Hamiltonian Eq. (15). Qubit persistence probability is often referred to as fidelity.

We proceed to obtain a double path integral expression for the qubit persistence probability Eq. (17), in the case of a qubit device interacting with its internal and external environment, represented by continuous fields.

If Q denotes, collectively, the coordinates q_μ of pertinent environmental degrees of freedom, the evolved state $|\psi_{qe}(Q_f)\rangle$ of the qubit device together with its environment at time t can be expressed by the following path integral:

$$|\psi_{qe}(Q_f)\rangle = \int dQ \int_Q^{Q_f} DQ e^{\frac{i}{\hbar}S[Q(t)]} |\psi_{qe}(Q)\rangle\,, \tag{18}$$

in which a standard integral is performed over all possible initial values Q of the coordinates, and the path integral is over all possible paths $Q(t)$. Also in Eq. (18), $S[Q(t)]$ denotes the total action, Q_f denotes the values $Q(t)$ at time t, and $|\psi_{qe}(Q)\rangle$ denotes the initial state of the qubit device together with its environment. In the absence of initial quantum correlations between the computational degrees of freedom and the environment, the combined initial state is given by the following tensor product:

$$|\psi_{qe}(Q)\rangle = |\psi_e(Q)\rangle \otimes |\mathrm{in}\rangle\,, \tag{19}$$

where $|\mathrm{in}\rangle$ is the initial state of the qubit, and $|\psi_e(Q)\rangle$ is the initial state of the environment. Equation (18) expresses the evolution of the environment explicitly, and that of the qubits implicitly. The evolved density matrix $\rho(Q_f, Q'_f)$ of the qubit device and environment is given by

$$\rho(Q_f, Q'_f) = |\psi_{qe}(Q_f)\rangle \left\langle \psi_{qe}(Q'_f) \right|. \tag{20}$$

If we substitute Eqs. (18) and (19) in Eq. (20), we obtain the following double path integral:

$$\begin{aligned} &\rho(Q_f, Q'_f) = \\ &\int dQ \int_Q^{Q_f} DQ e^{\frac{i}{\hbar}S[Q(t)]} |\psi_e(Q)\rangle \otimes |\mathrm{in}\rangle \int dQ' \int_{Q'}^{Q'_f} DQ' \langle \mathrm{in}| \otimes \langle \psi_e(Q')| e^{-\frac{i}{\hbar}S[Q'(t)]}, \end{aligned} \tag{21}$$

or equivalently,

$$\begin{aligned} &\rho(Q_f, Q'_f) = \\ &\int\int dQ dQ' \int_{Q'}^{Q'_f} \int_Q^{Q_f} DQ DQ' e^{\frac{i}{\hbar}S[Q(t)]} \left(|\psi_e(Q)\rangle \langle \psi_e(Q')|\right) \otimes \left(|\mathrm{in}\rangle \langle \mathrm{in}|\right) e^{-\frac{i}{\hbar}S[Q'(t)]}. \end{aligned} \tag{22}$$

The initial density matrix $\rho_{\mathrm{in}}(Q, Q')$ of the environment, only, is clearly

$$\rho_{\mathrm{in}}(Q, Q') = |\psi_e(Q)\rangle \langle \psi_e(Q')|\,. \tag{23}$$

Next substituting Eq. (23) in Eq. (22), one obtains

$$\begin{aligned} &\rho(Q_f, Q'_f) = \\ &\int\int dQ dQ' \int_{Q'}^{Q'_f} \int_Q^{Q_f} DQ DQ' e^{\frac{i}{\hbar}S[Q(t)]} \left(\rho_{\mathrm{in}}(Q, Q') \otimes |\mathrm{in}\rangle \langle \mathrm{in}|\right) e^{-\frac{i}{\hbar}S[Q'(t)]}. \end{aligned} \tag{24}$$

The bracketed tensor product in Eq. (24) is just the initial density matrix for the qubit together with its environment. Equation (24) expresses the total density matrix of the qubit device including its environment as a double path integral. We have been careful not to move the exponential on the far right to the left of the initial density matrix, to allow for the case in which the interaction term in the action contains an operator, such as a pseudospin matrix. This is the only way in which Eq. (24) differs from the standard form [13].

To obtain the reduced density matrix $\rho_q(t)$ of the qubits, which describes the computational state of the qubit device, we must trace over the environmental states at time t; thus,

$$\rho_q(t) = \int\int dQ_f dQ'_f \delta(Q_f - Q'_f) \rho(Q_f, Q'_f), \tag{25}$$

which expresses the trace for continuous degrees of freedom by means of the Dirac delta function. Next, substituting Eqs. (24) and (25) in Eq. (17), we obtain for the qubit persistence probability $P(t)$ the following expression:

$$\begin{aligned} P(t) = \int\int\int dQ_f dQ dQ' \int_{Q'}^{Q_f} \int_Q^{Q_f} DQ DQ' \langle \mathrm{fin}| e^{\frac{i}{\hbar}S[Q(t)]} |\mathrm{in}\rangle \\ \times \langle \mathrm{in}| e^{-\frac{i}{\hbar}S[Q'(t)]} |\mathrm{fin}\rangle \rho_{\mathrm{in}}(Q, Q'). \end{aligned} \tag{26}$$

The exponentials can be rewritten with

$$e^{\frac{i}{\hbar}S[Q(t)]} = e^{\frac{i}{\hbar}S_e[Q(t)]}\, T e^{-\frac{i}{\hbar}\int_0^t (H_q + H'[Q(t')])dt'}, \tag{27}$$

where $S_e[Q(t)]$ is the action of the environment, T denotes the time-ordered product, and $H'[Q(t')]$ expresses the dependence of the interaction on the environmental variables. For any Hermitian operator O, and its adjoint $O^\dagger$, one has

$$\langle \text{in}|\, O\, |\text{fin}\rangle = \langle \text{fin}|\, O^\dagger\, |\text{in}\rangle^* . \tag{28}$$

Substituting Eq. (27) in Eq. (26), and using Eq. (28), one obtains

$$\begin{aligned} P(t) = \int\int\int dQ_f dQ dQ' \int_{Q'}^{Q_f} \int_{Q}^{Q_f} DQ DQ' e^{\frac{i}{\hbar}(S_e[Q(t)] - S_e[Q'(t)])} \\ \times \langle \text{fin}|\, T e^{-\frac{i}{\hbar}\int_0^t (H_q + H'[Q(t')])dt'}\, |\text{in}\rangle \left(\langle \text{fin}|\, T e^{-\frac{i}{\hbar}\int_0^t (H_q + H'[Q'(t'')])dt''}\, |\text{in}\rangle \right)^* \\ \times \rho_{\text{in}}(Q, Q'). \end{aligned} \tag{29}$$

If we define the matrix element

$$A[Q(t)] \equiv \langle \text{fin}|\, T\, e^{-\frac{i}{\hbar}\int_0^t (H_q + H'[Q(t')])dt'}\, |\text{in}\rangle , \tag{30}$$

and the reduced density matrix propagator

$$J(Q_f, Q_f'; Q, Q') \equiv \int_{Q'}^{Q_f'} \int_{Q}^{Q_f} DQ DQ' e^{\frac{i}{\hbar}(S_e[Q(t)] - S_e[Q'(t)]} A[Q(t)] A^*[Q'(t)], \tag{31}$$

then we can rewrite Eq. (29) as

$$P(t) = \int\int\int dQ_f dQ dQ' J(Q_f, Q_f; Q, Q') \rho_{\text{in}}(Q, Q'). \tag{32}$$

We apply the qubit persistence probability formalism in Section 4 to a generic single-qubit device coupled to its thermal environment, and in Section 6 to a trapped-ion quantum register coupled to its ion vibrational modes.

4 DECOHERENCE IN QUBIT DEVICES

An illuminating calculation has been performed of decoherence of a single qubit, using a fully quantized approach, in which the environment is modeled as a continuum of thermal field modes [6,45,59]. Let us here sketch the calculation. The qubit density matrix without environmental interactions is of the same form as Eq. (3),

$$\begin{aligned} \rho^q(t) = \rho_{00}(t)\, |0\rangle \langle 0| + \rho_{01}(t)\, |0\rangle \langle 1| \\ + \rho_{10}(t)\, |1\rangle \langle 0| + \rho_{11}(t)\, |1\rangle \langle 1| , \end{aligned} \tag{33}$$

written in the binary computational basis. The total density matrix for the qubit, together with the modes of the thermal field environment to which it is coupled, is initially given by

$$\rho(0) = \rho^q(0) \otimes \rho^e = \sum_{i,j=0}^{1} \rho_{ij}(0)\, |i\rangle \langle j| \otimes \prod_k R_{kT}, \tag{34}$$

where ρ^e is the initial density matrix for the thermal field modes, and R_{kT} is the thermal density matrix of field mode k. The Hamiltonian H is taken to be

$$H = \frac{1}{2}\hbar\sigma_z\omega_0 + \sum_k \hbar b_k^\dagger b_k \omega_k + \sum_k \hbar\sigma_z (g_k b_k^\dagger + g_k^* b_k). \tag{35}$$

Here, the first term is the contribution of the free qubit, written in terms of pseudospin, represented by the z-component Pauli spin matrix σ_z, and $\hbar\omega_0$ is the energy separation between the two states of the qubit. The second term in Eq. (35) is the contribution of the environmental field modes alone for mode frequency ω_k, and $b_k^\dagger$ and b_k are mode creation and annihilation operators. The third term in Eq. (35) is the interaction term describing the interaction between the qubit and the thermal field modes, with coupling constants g_k. The corresponding evolution operator in the interaction picture is

$$U(t) = e^{-i\int_0^t \sum_k \sigma_z (g_k b_k^\dagger e^{i\omega_k t'} + g_k^* b_k e^{-i\omega_k t'})dt'}, \tag{36}$$

or equivalently,

$$U(t) = e^{\frac{1}{2}\sigma_z \sum_k (b_k^\dagger \xi_k(t) - b_k \xi_k^*(t))}, \tag{37}$$

where

$$\xi_k(t) \equiv 2(g_k/\omega_k)(1 - e^{i\omega_k t}). \tag{38}$$

It can be shown that the effect of the evolution operator Eq. (37) on a representative initial state is

$$[2^{-1/2}(|0\rangle + |1\rangle) \otimes |0_k\rangle] \xrightarrow{U(t)} 2^{-1/2}\left(|0\rangle \otimes \left|-\frac{1}{2}\xi_k(t)\right\rangle + |1\rangle \otimes \left|\frac{1}{2}\xi_k(t)\right\rangle\right). \tag{39}$$

Here, $|0_k\rangle$ is the vacuum state for a free field mode, and $\left|\pm\frac{1}{2}\xi_k\right\rangle$ are coherent states of amplitude $\pm\frac{1}{2}\xi_k$. Equation (39) shows how the qubit state becomes entangled with the environment. Also, it can be shown that the overlap between the different environmental field states decreases with time, resulting in orthogonaliztion of the environmental basis states, and qubit decoherence, as discussed generically in Section 2.

The total density matrix $\rho(t)$ at time t is given by

$$\rho(t) = U(t)\rho(0)U(t)^{-1}, \tag{40}$$

expressed in terms of the evolution operator Eq. (37). The reduced density matrix $\rho_q(t)$ of the qubit at time t is

$$\rho_q(t) = \mathrm{Tr}_R(U(t)\rho(0)U(t)^{-1}), \tag{41}$$

where the trace is taken over the thermal environmental field modes. The coherences and the populations were calculated for a one-dimensional field in the low-temperature limit. The populations are unaffected by decoherence for the case at hand, namely,

$$\langle 0|\,\rho_q(t)\,|0\rangle = \rho_{00}(0), \quad \langle 1|\,\rho_q(t)\,|1\rangle = \rho_{11}(0). \tag{42}$$

The coherences become

$$\langle 1|\,\rho_q(t)\,|0\rangle = e^{-\Gamma(t)}\rho_{10}(0), \quad \langle 0|\,\rho_q(t)\,|1\rangle = e^{-\Gamma(t)}\rho_{01}(0), \tag{43}$$

where the decoherence function $\Gamma(t)$ is defined by [6]

$$\Gamma(t) = \frac{1}{2}\ln(1 + \omega_c^2 t^2) + \ln\left[\left(\frac{\beta\hbar}{\pi t}\right)\sinh\left(\frac{\pi t}{\beta\hbar}\right)\right]. \tag{44}$$

Here, $\beta = 1/kT$, where T is the temperature. Also, ω_c is a cutoff frequency corresponding to some characteristic length scale of the system, below which the qubit-environment coupling is negligible (such as the Debye frequency for a phonon field).

Let us proceed by calculating the qubit persistence probability for the following initial state:

$$|\mathrm{in}\rangle = 2^{-1/2}(|+\rangle + |-\rangle), \tag{45}$$

in which

$$|+\rangle \equiv |1\rangle, \quad |-\rangle \equiv |0\rangle, \quad \sigma_z\,|\pm\rangle = \pm\,|\pm\rangle. \tag{46}$$

In the interaction picture, the unperturbed state of the qubit at time t remains identical to $|\text{in}\rangle$; hence

$$|\text{fin}\rangle = 2^{-1/2}\left(|+\rangle + |-\rangle\right). \tag{47}$$

The qubit persistence probability Eq. (17) is then given by

$$P(t) = \langle \text{fin}|\, \rho_q(t)\, |\text{fin}\rangle\,. \tag{48}$$

Using Eqs. (41) to (43), and (45) to (48), one obtains

$$P(t) = \frac{1}{2} + \frac{1}{2}e^{-\Gamma(t)}. \tag{49}$$

Substituting Eq. (44) in Eq. (49), and performing a Taylor series expansion in the time, we find that

$$P(t) = 1 - \left(\frac{t}{\tau}\right)^2 + ..., \tag{50}$$

where

$$\tau = \frac{2}{\omega_c}. \tag{51}$$

The short-time scale, τ, for the decoherence is seen to be of the order of the inverse cutoff frequency (such as the Debye period for a phonon field) corresponding to a characteristic length scale below which the qubit-environment coupling is negligible. The model has also been extended to analyze quantum decoherence in a quantum register (see next Section) of N qubits [6,45,59]. For independent interaction of the qubits with the environment, the decoherence time scales as $1/N$, while for collective interaction it scales as $1/N^2$.

5 QUANTUM REGISTER

One of the main ingredients of any sizable quantum computer would be the quantum register. In Section 6, a review of a detailed calculation of a particular type of quantum decoherence in a quantum register will serve as an example of the intricacies and difficulties which can be involved in the calculation of quantum decoherence. We first review the characteristics of a quantum register. A quantum register may be thought of as a row of N qubits. A binary number,

$$n = \sum_{k=0}^{N-1} n_k 2^k, \qquad n_k = 0 \text{ or } 1 \tag{52}$$

can be stored in the quantum register, and is represented by a product state of the N qubits, namely,

$$|n\rangle = |n_{N-1}\rangle\, |n_{N-2}\rangle \cdots |n_1\rangle\, |n_0\rangle\,. \tag{53}$$

Here, tensor products are implicit, and the order of the kets corresponds to the order of the qubits in the register. Each ket in the product corresponds to a single qubit. A general state $|\psi\rangle$ of the free quantum register is given by the N-qubit entangled state,

$$|\psi\rangle = \sum_{n=0}^{2^N-1} \alpha_n\, |n\rangle\,, \tag{54}$$

where the α_n are complex numbers, and the sum is over all 2^N possible product Boolean states, Eq. (53), thereby forming a 2^N dimensional Hilbert space. For example, a three-bit classical register can store only one of eight different numbers, {000,001,010,011,100,101,110,111}, using a binary representation of numbers between 0 and 7. But a quantum register consisting of three qubits could store up to eight numbers at the same time in a quantum superposition. The state of the three-qubit quantum register is in general given by

$$\begin{aligned} |\psi\rangle = {} & \alpha_{000}\,|0\rangle\,|0\rangle\,|0\rangle + \alpha_{001}\,|0\rangle\,|0\rangle\,|1\rangle + \alpha_{010}\,|0\rangle\,|1\rangle\,|0\rangle + \alpha_{011}\,|0\rangle\,|1\rangle\,|1\rangle \\ & + \alpha_{100}\,|1\rangle\,|0\rangle\,|0\rangle + \alpha_{101}\,|1\rangle\,|0\rangle\,|1\rangle + \alpha_{110}\,|1\rangle\,|1\rangle\,|0\rangle + \alpha_{111}\,|1\rangle\,|1\rangle\,|1\rangle\,. \end{aligned} \tag{55}$$

This corresponds to a coherent superposition of the numbers from 0 to 7. For the N-qubit quantum register, although measuring the register's contents will yield only one number, a quantum computation could effectively manipulate all 2^N numbers at once. For example, if each qubit consists of two states of an atom, tuned laser pulses could perform operations on the electronic states so that an initial superposition of 2^N numbers could evolve into a different superposition. This evolution results in massive parallelism, since each number in the superposition is affected.

To prepare a specific number in a quantum register, N elementary operations must be performed. Each of the N qubits must be put in one of two states. N elementary operations, which can be represented by unitary transformations on each qubit, can prepare the register in a coherent superposition of 2^N numbers, to be stored in the register. This process can be seen as follows. One can represent the Boolean states $|0\rangle$ and $|1\rangle$ by the vectors

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{56}$$

If, for example, the first qubit of a three-qubit register is in the state $|0\rangle$, then applying a Hadamard operator H, causes the state of the qubit to become

$$H|0\rangle = 2^{-1/2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 2^{-1/2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 2^{-1/2} (|0\rangle + |1\rangle), \tag{57}$$

which is an equally weighted superposition of Boolean states $|0\rangle$ and $|1\rangle$. The Hadamard operator applied to each qubit of a three-qubit quantum register, each initially in state $|0\rangle$, yields the state,

$$\begin{aligned} |\psi\rangle = \prod_{i=1}^{3} (H|0\rangle) = H|0\rangle H|0\rangle H|0\rangle &= 2^{-3/2} (|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle) \\ &= 2^{-3/2} (|0\rangle + |1\rangle)(|0\rangle|0\rangle + |0\rangle|1\rangle + |1\rangle|0\rangle + |1\rangle|1\rangle) \\ &= 2^{-3/2} (|0\rangle|0\rangle|0\rangle + |0\rangle|0\rangle|1\rangle + |0\rangle|1\rangle|0\rangle + |0\rangle|1\rangle|1\rangle \\ &\quad + |1\rangle|0\rangle|0\rangle + |1\rangle|0\rangle|1\rangle + |1\rangle|1\rangle|0\rangle + |1\rangle|1\rangle|1\rangle), \end{aligned} \tag{58}$$

in which the order of the kets is preserved and corresponds to the order of the qubits in the register. Next, using a notation for states representing numbers to the base 10, that is,

$$\begin{aligned} &|0\rangle \equiv |0\rangle|0\rangle|0\rangle, \quad |1\rangle \equiv |0\rangle|0\rangle|1\rangle, \quad |2\rangle \equiv |0\rangle|1\rangle|0\rangle, \quad |3\rangle \equiv |0\rangle|1\rangle|1\rangle, \\ &|4\rangle \equiv |1\rangle|0\rangle|0\rangle, \quad |5\rangle \equiv |1\rangle|0\rangle|1\rangle, \quad |6\rangle \equiv |1\rangle|1\rangle|0\rangle, \quad |7\rangle \equiv |1\rangle|1\rangle|1\rangle, \end{aligned} \tag{59}$$

one can rewrite Eq. (58) as

$$|\psi\rangle = 2^{-3/2} (|0\rangle + |1\rangle + |2\rangle + |3\rangle + |4\rangle + |5\rangle + |6\rangle + |7\rangle) = 2^{-3/2} \sum_{n=0}^{2^3-1} |n\rangle. \tag{60}$$

Thus, more generally, if an N-qubit register is initially in the state $|0\rangle|0\rangle|0\rangle \ldots |0\rangle$, one can apply the Hadamard operator H, to each qubit, and the resulting state of the register is an equally weighted superposition of all 2^N numbers, namely,

$$|\psi\rangle = \prod_{i=1}^{N} (H|0\rangle) = H|0\rangle H|0\rangle \ldots H|0\rangle = 2^{-N/2} \sum_{n=0}^{2^N-1} |n\rangle. \tag{61}$$

The N elementary operations generate a state containing all 2^N possible numerical values of the register. This provides a method for generating an important intermediate state in Shor's quantum factoring algorithm. If the quantum register is prepared in such a coherent superposition of numbers, and all subsequent computational transformations are unitary (preserving the superposition of states), then in each step, the computation is performed on each of the numbers in the superposition simultaneously. In Section 6, following an introduction to the trapped-ion approach to quantum computer development, the physics is quantitatively addressed of one possible mechanism of quantum decoherence in a particular implementation of a quantum register, namely, a trapped-ion quantum register.

6 ION-TRAP QUANTUM COMPUTER

In an early version [60] of an ion-trap quantum computer [56,59–64], a one-dimensional lattice of identical ions is stored and laser cooled in a linear Paul trap (radio frequency quadrupole trap). The linear array of ions acts as a quantum register (see previous section). The radio-frequency trap potential strongly confines the ions radially about the trap axis, and an electrostatic potential causes the ions to oscillate along the trap axis in an effective harmonic potential. Laser cooling results in localization of the ions along the trap axis, with spacing determined by Coulomb repulsion and the confining axial potential. The lowest frequency mode of collective oscillation of the ions is the axial center-of-mass mode, in which all the ions oscillate together with the same phase. Each of the trapped ions acts as a qubit, in which the two pertinent states are the electronic ground state and a long-lived excited state. By means of coherent interaction of a precisely controlled laser pulse with any one of the ions in a standing wave configuration, one can manipulate the ion's electronic state and the quantum state of the collective center of mass mode of the oscillator. In principle, the center of mass mode can then be used as a bus, quantum dynamically connecting the qubits, to implement the necessary quantum logic gates. The general state of the line of ions comprising the quantum register is an entangled linear superposition of their states. A completed computation can be read out by a quantum jump measurement technique. Experimental demonstration of the ion-trap approach began with state preparation, quantum gates, and measurement for a single trapped ion [48]. Later, coherent Rabi flopping and a controlled-not gate between spin and motional qubits were demonstrated [65]. Also, selected single-spin qubit operations have been demonstrated on a chain of ions [66]. A two-spin qubit gate and a controlled-NOT gate were demonstrated [67,68]. Also a four-spin maximally entangled state has been produced [67]. Two-logical-qubit operations have not yet been performed. A number of experimental and theoretical issues regarding the ion-trap approach to quantum computation have been explored [59,61–63,68–81]. An experimental difficulty in implementing this approach is cooling the ions to the ground state in the trap. An important source of decoherence is apparently the heating due to coupling between the ions and noise voltages in the trap electrodes [70]. The speed of an ion trap quantum computer would apparently be limited by the frequencies of vibrational modes in the trap. It has been contemplated that 100 quantum gate operations could be applied to a few ions [69]. It is doubtful that sufficient storage capacity and coherence will ever be achieved to enable factoring of hundred-digit numbers by the trapped-ion approach; however, it is contemplated that scalability to large numbers of qubits might be achieved by interconnecting ion-trap arrays [82,83]. Also, it is possible that smaller traps can be constructed using nanofabrication technology.

Some time ago, an intricate path integral calculation was performed of quantum decoherence in a trapped-ion quantum register, arising from vibrational coupling of the ions [84]. The calculation, which is sketched in the following, illustrates the complicated nature of quantum decoherence for just one of many possible decoherence mechanisms. A trapped-ion quantum register consists of a linear array of N identical ions confined in a linear radio-frequency quadrupole trap. To each ion corresponds one ket in the tensor product, Eq. (53). Each ion contains one qubit consisting of a ground state $|g\rangle_i$ and an excited state $|e\rangle_i$ of the ion. It is convenient to employ the following notation:

$$|+\rangle_i \equiv |e\rangle_i\,, \qquad |-\rangle_i \equiv |g\rangle_i\,, \qquad \sigma_{iz}\,|\pm\rangle_i = \pm\,|+\rangle_i\,, \tag{62}$$

where σ_{iz} is the z component of pseudospin for the ith ion. The following initial state for the quantum register was addressed:

$$|\mathrm{in}\rangle = \prod_{i=1}^{N} 2^{-1/2}\,(|+\rangle_i + |-\rangle_i)\,. \tag{63}$$

This state has the same structure as Eq. (61). Here, each ion is in a superposition of its ground and excited states. Such an N-fold tensor product state is typical in quantum computation, and can, in principle, be generated by the use of appropriate distributions of laser pulses to excite or de-excite the ionic qubits. The coupling between ion vibrations in the confining electromagnetic trap potential and the qubit states of the ions is one of the many possible processes contributing to decoherence in such a qubit device, and is the decoherence process addressed here. The charged ion vibrations create fluctuating electric fields that drive transitions between the atomic states representing the qubits, and alter the time evolution of the computational degrees of freedom. Even at zero temperature, zero-point ionic motion can cause decoherence. The total Hamiltonian H for the ion-trap quantum

register, together with its environment of ionic vibrations, is given by

$$H = H_i + H_{nm} + H', \tag{64}$$

where H_i is the free qubit Hamiltonian having eigenstates that are N-fold tensor products of $|g\rangle_i$ and $|e\rangle_i$, H_{nm} is the Hamiltonian of the normal modes of vibration of the ions, and H' is the interaction term. Specifically, one has

$$H_i = \frac{1}{2}\sum_i \hbar\omega_0\sigma_{iz}, \tag{65}$$

$$H_{nm} = \sum_\mu \left(\frac{p_\mu^2}{2m} + \frac{1}{2}m\omega_\mu^2 q_\mu^2\right), \tag{66}$$

and

$$H' = \sum_{i,\mu}\left(\sigma_{ix}c_{i\mu x} + \sigma_{iy}c_{i\mu y}\right)q_\mu. \tag{67}$$

In Eq. (65), $\hbar\omega_0$ is the energy difference between the states $|g\rangle_i$ and $|e\rangle_i$, and a pseudospin representation of the two-level atom is employed, in which σ_{iz} is the z-component Pauli spin matrix corresponding to the ith ion. In Eq. (66), m is the mass of each ion, and q_μ and p_μ are the ion-vibrational normal mode coordinates and momenta, respectively, for mode μ with frequency ω_μ. In Eq. (67), σ_{ix} and σ_{iy} are the x and y component Pauli spin matrices, respectively, corresponding to the ith ion, and $c_{i\mu x}$ and $c_{i\mu y}$ are the transverse coupling constants. An approximation can be made based on the following inequalities, with $c_{i\mu} \equiv (c_{i\mu x}^2 + c_{i\mu y}^2)^{1/2}$:

$$\omega_0 \gg \omega_\mu, \quad \hbar^{-1}\left\langle\sum_\mu c_{i\mu}q_\mu\right\rangle \ll \omega_\mu, \quad \hbar^{-2}\sum_\mu \langle c_{i\mu}q_\mu\rangle^2/\omega_0 \ll \omega_\mu, \tag{68}$$

which are likely to be satisfied for vibrational frequencies small relative to atomic transition frequencies, sufficiently small couplings, and for vibrational states cooled to near zero temperature. The particular case was addressed of Ba^+ ions with qubit states $|g\rangle_i$ and $|e\rangle_i$ lying in $6s^2S_{1/2}$ and $5d^2D_{5/2}$ multiplets, respectively, with $\Delta M = \pm 1$. The decay $^2D_{5/2} \rightarrow ^2S_{1/2}$ is an $E2$ process with spontaneous emission time $\tau_{sp} \simeq 35$ s, and with $\omega_0/2\pi = 1.7 \times 10^{14}$Hz. One obtains the following expression for the magnitude of the transverse coupling constants:

$$c_{i\mu} \equiv c_{ir\alpha} = -q\sum_{j\neq i}\left(F_r^i - F_r^j\right)(z_i - z_j)^{-4}\left|\langle e|\,\widehat{Q}_{\alpha z}^i\,|g\rangle\right|, \tag{69}$$

in which the mode index $\mu = (r,\alpha)$ corresponds to the α-component of mode r, where r ranges over the number of vibrational modes, and $\alpha = x, y$, since $|\Delta M| = 1$. Also in Eq. (69), q is the ion charge, F_r^i are normal mode eigenvectors, z_i is the equilibrium position of the ith ion, and $\widehat{Q}_{\alpha z}^i$ is the quadrupole moment tensor for the ith ion. The transition matrix element $\langle e|\,\widehat{Q}_{\alpha z}^i\,|g\rangle$, expressed in terms of the spontaneous emission time, is given by

$$\left|\langle e|\,\widehat{Q}_{\alpha z}^i\,|g\rangle\right|^2 = \left(\frac{18\hbar c^5}{\omega_0^5}\right)\frac{1}{\tau_{sp}}, \tag{70}$$

where c is the vacuum velocity of light.

The starting point for the decoherence calculation [84] is Eq. (32) above, in which the initial density matrix $\rho_{\rm in}(Q,Q')$ of the environment, consisting of the modes of vibration of the ions, is taken to be thermal with temperature T, and given by

$$\rho_{\rm in}(Q,Q') = \rho_0 e^{-\beta H_{nm}}, \tag{71}$$

where ρ_0 is a constant, and $\beta = 1/kT$. Using Eq. (71) together with a perturbative adiabatic approximation based on the inequalities Eq. (68), the double path integral in Eqs. (31) and (32) can be reduced to obtain the following expression for the qubit persistence probability for the ion-trap quantum register:

$$P(t) = 2^{-2N}\sum_{\{s,s'\}}\prod_\mu\left(\sinh(\beta\hbar\omega_\mu/2)\left[\sinh\left((\beta\hbar\omega_\mu - i\delta_\mu t)/2\right)\right]^{-1}\right), \tag{72}$$

where N is the number of ions in the register, δ_μ is defined by

$$\delta_\mu \equiv (m\hbar\omega_0\omega_\mu)^{-1} \sum_i (s_i - s_i')c_{i\mu}^2, \tag{73}$$

and

$$\{s, s'\} \equiv \{s_1, s_2, ...s_N; s_1', s_2', ...s_N'\}, \quad s_i = \pm 1, \quad s_j' = \pm 1. \tag{74}$$

Expanding Eq. (72) as a Taylor series in time, one obtains

$$P(t) = 1 - \left(\frac{t}{\tau_d}\right)^2 + ..., \tag{75}$$

in which the decoherence time τ_d is given by

$$\tau_d = \\ 2\omega_0 \left\{ \sum_i \left[\left(\sum_\mu (m\hbar\omega_\mu)^{-1} c_{i\mu}^2 \coth(\beta\hbar\omega_\mu/2) \right)^2 + \sum_\mu (m\hbar\omega_\mu)^{-2} c_{i\mu}^4 \mathrm{csch}^2(\beta\hbar\omega_\mu/2) \right] \right\}^{-1/2}. \tag{76}$$

In the zero temperature limit, $\beta \to \infty$, Eq. (72) becomes

$$P(t) = \prod_i \cos^2 \left(\left[\sum_\mu (2m\hbar\omega_0\omega_\mu)^{-1} c_{i\mu}^2 \right] t \right), \tag{77}$$

and the decoherence time becomes

$$\tau_d = 2m\hbar\omega_0 \left(\sum_i \left(\sum_\mu \omega_\mu^{-1} c_{i\mu}^2 \right)^2 \right)^{-1/2}. \tag{78}$$

By using a continuous approximation for the distribution of ions along the linear array, and replacing all the mode frequencies by the frequency ω_{tN} of the zigzag transverse mode, which is the smallest, one obtains the following lower bound on the decoherence time:

$$(\tau_d/\tau_{sp}) > 2.8 \left[\ln(0.8N)\right]^{8/3} N^{-35/6} q^{10/3} m^{-5/3} c^{-5} \omega_0^6 \omega_{tN} \omega_z^{-16/3}, \tag{79}$$

where

$$\omega_{tN} \approx \left[\omega_t^2 - (9/16)\zeta(3) N^2 (\ln(0.8N))^{-1} \omega_z^2 \right]^{1/2}, \tag{80}$$

in which ω_z and ω_t are the longitudinal and transverse center of mass vibration frequencies, respectively, and the value of the Riemann zeta function is $\zeta(3) \approx 1.2$. In Eqs. (79) and (80), note the strong inverse dependence of the decoherence time of the quantum register on the number N of qubits in the register. One can obtain the critical transverse center of mass vibration frequency ω_{tcr}, the threshold of the zigzag instability, by setting ω_{tN} to zero in Eq. (80):

$$\omega_{tcr} = (3/4)(\zeta(3))^{1/2} (\ln(0.8N))^{-1/2} N\omega_z. \tag{81}$$

Setting $\omega_{tN} = \omega_{tcr}/2$ in Eq. (79), one obtains (τ_d/τ_{sp}) in the 10^4 to 10^8 range for $(\omega_0/2\pi) = 1.7 \times 10^{14}$ Hz, $(\omega_z/2\pi)$ in the 10^4 to 10^5 Hz range, and $N = 1000$. Although this result might be considered encouraging for ion-trap qubit devices, the result of other work is pessimistic [59,78–81].

Some other possible sources of quantum decoherence in an ion-trap quantum computer include ion heating due to electrode voltage noise, spontaneous emission, unwanted atomic transitions, noise in the center-of-mass mode, laser instabilities, electromagnetic field fluctuations, laser intensity fluctuations, collisions with background gas in the vacuum chamber, phonon scattering, thermal radiation, and ion cross-talk. Including only the effect of spontaneous emission without quantum error correction, it has been argued that the largest number one can factorize is really small [59]. It is noteworthy that quantum decoherence of a generic quantum register of arbitrary length, coupled to an environment of arbitrary coherence length, shows a strong dependence on the input state [85]. The effects of decoherence on a quantum register may be lessened by encoding quantum information in decoherence-free subspaces in which the states are invariant under environmental coupling [62,86].

7 OPTICAL QUANTUM COMPUTERS

Optical quantum computers are of three general types, based on (1) ordinary linear optics, (2) nonlinear optics, and (3) probabilistic linear optics with conditional logic. In a quantum computer based on ordinary linear optics, a basic qubit consists of two path states of a single photon [1,87]. A photon leaving an optical element, such as a beam splitter with two exit ports, has a propensity to exit along either path, so the photon becomes a two path-state system. Qubits can also be based on the photon-polarization state. The linear optical elements composing the device include beam splitters, mirrors, polarizers, wave plates, etc. The initial state of the device need only consist of a single photon entering the device at a beam splitter. All the necessary quantum gates can be implemented. Even two-qubit gates such as the controlled-NOT gate can be implemented. By cascading the number of beam splitters, locating one at each alternative path in a network of optical elements, the device becomes a multiple-qubit system. However, because of the resulting exponential proliferation of optical elements needed to form a large number of path qubits, such a linear-optical quantum computer is limited to a relatively small number of qubits. The device is therefore limited to implementing small-scale quantum networks of optical elements for performing small quantum algorithms, such as those involved in simple quantum error correction and quantum teleportation.

The second type of optical quantum computer is a multi-photon device employing nonlinear optical elements [56]. Nonlinear optical elements are needed so that the state of one photonic qubit can control the state of another at certain nodes in the network. The exponential cascading required by large linear-optical quantum computers is circumvented. The problem with the use of traditional nonlinear optical elements for implementing conditional dynamics, in which the state of one photon conditionally modulates the state of another, is the prohibitively huge nonlinear susceptibility required to produce the necessary phase shifts at the two-photon level of intensity. Practical nonlinear photon gates operating at the two-photon level of intensity are not presently available; and various innovative approaches have not been successful. It should however be noted that recent work on controlled slow light may facilitate innovative nonlinear optical approaches [88–90].

Another type of optical quantum computer would use single photon sources, passive linear optics, fast efficient photodetectors, and fast electrooptic feed forward [91,92]. Two qubit gates would not require huge nonlinearities, but would instead exploit conditional logic based on selected photodetector responses. In this approach, the success of gate operations is intrinsically probabilistic. Nondeterministic quantum logic operations have been demonstrated [93]. Using linear optics together with spontaneous parametric down conversion and post selection, three-photon Greenberger-Horne-Zeilinger entanglement has been observed [94], and it may be that this can also be accomplished with single photon sources. Practical fast and reliable on-demand single photon sources and efficient discriminating single-photon detectors are not yet available. Although photonic qubits offer greater immunity to decoherence, this is offset by the intrinsic probabilistic nature of this approach. Also, the fidelity of all optical components and detectors must be significantly improved. Although decoherence is not an obstacle to the development of a reasonably small special-purpose optical quantum information processor, it would become an issue in any attempt to scale up the device to include large numbers of optical elements. Some possible sources of decoherence include photon losses, imperfect optical elements, ambient light, imperfect interferometric paths, and imperfect mode matching.

8 CAVITY QED QUANTUM COMPUTER

Another early approach to the development of a quantum computer was offered by cavity quantum electrodynamics (QED) [56,59,62,63,95–99]. In one cavity QED approach, a number of neutral atoms are trapped inside a high-finesse optical cavity [100]. Electronic states of the atom act as qubits to store information. The atoms in the cavity interact with a quantized mode of the cavity. The separations between the atoms are much greater than the wavelength of the cavity mode, and the atoms can interact individually with laser pulses. This permits

sequences of operations between two qubits and the implementation, in principle, of an entire quantum network. The qubits may consist of ground state levels of the trapped atoms. Quantum gates can be implemented by the atoms being coupled to individual laser pulses and entangled by exchange of a cavity photon. Pulsed lasers can be used to drive transitions in one atom conditionally on the internal states of another atom. Also, the polarization states of a photon can serve as a qubit. An atom trapped in the cavity can act as an effective nonlinear medium to mediate interactions between two photons, and thereby implement a two-photon quantum gate, in which the polarization state of one photon alters the phase of the other photon [50]. Letting $|l\rangle_i$ and $|r\rangle_i$ denote left and right circular polarization states of photon i ($i = 1, 2$), one has, effectively, $|l\rangle_1 |l\rangle_2 \Longrightarrow |l\rangle_1 |l\rangle_2$, $|l\rangle_1 |r\rangle_2 \Longrightarrow e^{i\phi_2} |l\rangle_1 |r\rangle_2$, $|r\rangle_1 |l\rangle_2 \Longrightarrow e^{i\phi_1} |r\rangle_1 |l\rangle_2$, and $|r\rangle_1 |r\rangle_2 \Longrightarrow e^{i(\phi_1+\phi_2+\Delta)} |r\rangle_1 |r\rangle_2$, where ϕ_1 and ϕ_2 are differential phases between the two polarization states, and Δ is the conditional phase shift. These transformations, are accomplished first by one photon being stored in the cavity, in which the right circular polarization state couples strongly to the atom, but the left circular polarization state does not. Next, another photon traverses the cavity, also interacting preferentially in one polarization state with the atom, and acquiring the conditional phase shift only if the photons are in the right circular polarization state. Thus, the phase shift is conditional on the polarization state of both photons; the result is a two-qubit quantum logic gate. The gate exploits the extremely large optical nonlinearities that are achievable in cavity QED. The cavity may operate in a moderate-coupling parameter regime in which $\kappa > (g^2/\kappa) > \gamma$, or even the strong coupling regime in which $g \gg (\kappa, \gamma)$, where κ is the cavity-field damping rate, g is the dipole coupling rate of the atom to the cavity, and γ is the transverse atomic decay rate to noncavity modes. In the moderate coupling regime, the coherent coupling of the atom to the cavity mode (at rate g^2/κ) dominates incoherent emission into free space (at rate γ). This enables strong coupling of a single atom to the cavity mode, allowing efficient transfer of electromagnetic fields from input to output channels (at rate κ).

Conditional dynamics at the single quantum level has also been achieved with single atoms interacting with very weak microwave fields in superconducting cavities [51]. Atomic wave function phase shifts were produced by microwave fields with, on average, much less than one photon in the cavity. In related work, a quantum memory was implemented in which the quantum information carried by a two-level atom was transferred to a cavity, and subsequently to another atom [101]. Within the same framework, a methodology was developed for the construction of arbitrary quantum computational networks with all the necessary quantum gates to perform all quantum logic operations [102,103]. In cavity QED, sources of decoherence include spontaneous emission from excited states of atoms, cavity decay during gate operation, laser fluctuations, and ambient light. Maintaining coherence between multiple cavities is problematic. Also, the trapping and localization of atoms inside cavities present formidable difficulties. Possible scaling up of the cavity-QED approaches to more than several qubits remains to be accomplished and poses serious problems, which may limit the practical utility of cavity-QED quantum computers to special-purpose small-scale quantum information processing (for use in quantum communication, for example). However, it is possible that the cavity-QED paradigm can be implemented using much smaller nanofabricated circuits.

Cavity QED is also being implemented in the development of possible quantum computer communication networks. In the cavity-QED approach to quantum information processing, both the states of atoms confined in cavities and the states of photons interacting with the atoms may serve as qubits to store and transfer quantum information. Although the difficulties in the successful trapping and localization of atoms inside high-finesse optical cavities are considerable (perhaps making the development of large-scale universal quantum computers based on the cavity-QED concept an unattainable goal), the development of small-scale special-purpose quantum information processors involving limited numbers of trapped atoms will likely be possible. For example, cavity-QED can provide a practical approach to the development of controlled single-photon sources, the synthesis of entangled states, and quantum teleportation between cavities. Both photonic and atomic qubits may be exploited with the cavity-QED paradigm, in the form of quantum information networks that enable the implementation of quantum communication protocols and distributed quantum computation [59,62,99,104–108]. Multiple atom-cavity systems located at distant network nodes may be interconnected with optical fibers, or perhaps even use free-space transmission. Analysis has been performed of basic network operations, including local quantum information processing, quantum state transmission between network nodes, and quantum entanglement distribution [104–110]. Ideal transmission may be permitted after a finite number of trials, without disturbing the quantum information. Possible sources of decoherence include absorption of photons in the optical fibers and cavity

mirrors, cavity and laser design errors, ambient light, and scattering.

9 NMR QUANTUM COMPUTER

Proof of principle of quantum computation was first accomplished by still another approach, which makes innovative use of established nuclear magnetic resonance (NMR) technology. NMR is used as the basis for quantum computation when certain liquids are used along with available NMR instrumentation [56,59,62-64,99,111-114]. The qubits are the spins of atomic nuclei in the molecules constituting the liquid. These qubits are extremely well isolated from their environment and have long decoherence times. The nuclear spin orientations in a single molecule form a quantum data register. The liquid contains about 10^{23} molecules at room temperature and undergoes strong random thermal fluctuations. The liquid is located in a large magnetic field, and each spin can be oriented either in the direction of the magnetic field ($|\uparrow\rangle = |0\rangle$) or opposite ($|\downarrow\rangle = |1\rangle$). An NMR quantum computer operating on N qubits uses molecules having N atoms with distinguishable spins in the frequency domain. The input to the computer is an ensemble of nuclear spins initially in thermal equilibrium. Each spin can be manipulated with resonant rf pulses, and the coupling between neighboring nuclear spins can be exploited to produce quantum logic gates. The spins have scalar coupling, and a driving pulse in resonance can tip a spin conditional on the state of another spin, thus providing a quantum bus channel. A sequence of rf pulses and delays produces a series of quantum logic gates connecting the initial state to a desired final state. By suitable timing of each pulse, a desired unitary transformation can be resonantly performed on a single spin of a molecule even though all the spins in the molecule are exposed, since they all have slightly different resonant frequencies. The decoherence times of the spins are long enough that the qubits can be stored for a sufficiently long time. The average magnetic moment of all the nuclei together is big enough to produce a detectable magnetic field for measurement purposes. The liquid consists effectively of a statistical ensemble of single-molecule quantum computers, which can be described by a density matrix. The method exploits the structure present in thermal equilibrium to produce a perturbation in the system's large density matrix that is effectively equivalent to a pure state of much smaller dimension, a pseudo-pure state. The system of molecules, each having N nuclear spins, can be described by a density matrix $\rho = 2^{-N} I + \rho_\Delta$ [111–113] in which the first term describes an equilibrium part that is proportional to the identity I, and the second term ρ_Δ is a traceless matrix representing the deviation from equilibrium. For an appropriate pulsed field sequence, the deviation transforms as a density matrix, the deviation density matrix, and represents the statistical ensemble of single-molecule quantum computers in the form of a bulk effective quantum computer. An effective pure state can be distilled out of ρ_Δ by means of a data compression pulse sequence. An appropriate computational procedure yields a deterministic result in which measuring the ensemble yields a nonvanishing average. Readout is performed by measurement of the magnetization of the bulk sample. This is bulk quantum computation employing large ensembles of quantum systems instead of single systems. Such a bulk quantum computer acts as an ensemble of many small quantum computers carrying out computations independently in parallel. The initial state of each is random, and only ensemble averages of each computer register can be measured. The ensemble can effectively behave like a pure state, since even if, for example, only a small fraction of the systems are in their ground states, the ones that are not can be arranged so that their signals cancel each other, and only the fraction in the ground state produces a nonvanishing signal, making the ensemble appear to be pure. Generally, if a chosen fraction of the states can be labeled, and the rest caused to average away, then an effective pure state can be produced.

Some experimental accomplishments to date in the NMR approach include: implementation of Grover's fast quantum search algorithm for a system with only four states, demonstrations of two- to seven-qubit quantum computation, proof-of-principle quantum error correction, implementation of a quantum algorithm determining whether an unknown function is constant or has value 0 for half its arguments and 1 for the rest, implementation of a quantum algorithm for estimating the number of matching items in a search operation, implementation of Shor's algorithm in factoring a two-digit number, and quantum teleportation [63].

The NMR quantum computers have poor scaling with the number of qubits. The measured signal scales

as 2^{-N}. This feature will likely limit liquid NMR quantum computers to applications requiring only about 10 qubits. Attempts are underway to go beyond 10 qubits by developing solid-state NMR devices [63], however this approach would likely be limited to less than 30 qubits.

10 SOLID STATE QUANTUM COMPUTERS

Since it is unlikely that NMR quantum computers can be scaled up to produce large-scale quantum computations involving very large numbers of qubits, a popular hybrid concept was proposed that would use semiconductor physics to deterministically manipulate nuclear spins [115,63]. The silicon-based nuclear spin quantum computer would consist of an equally spaced linear or planar array of dopant phosphorus nuclear spins implanted in a silicon semiconductor crystal, separated by an insulator layer from overlaying voltage-controlled metal gate electrodes. The gate electrodes would implement quantum logic operations by affecting the shape of the electron wave function surrounding each phosphorus nucleus. The qubits would be the nuclear spins of the phosphorus nuclei embedded periodically in the silicon crystal, each located directly beneath a gate referred to as an 'A gate'. A phosphorus atom in a silicon host is an electron donor, and at room temperature one of its outer electrons can move freely in the crystal; however, at the very low temperature of operation of the device, the electron is weakly bound by the phosphorus ion, and the electron spin can interact with the nuclear spin. Thus, the weakly bound electron spin could affect the state of a qubit, since electron and nuclear spins are coupled by the hyperfine interaction. Also, the electrons could mediate nuclear spin interactions and facilitate the measurement of nuclear spins. A voltage applied to an A gate could cause the wave function of the electron bound to the phosphorus nucleus beneath it to become altered, thereby changing its overlap with the nucleus. This electron-nucleus interaction would affect the relative energies of the nuclear qubit, and therefore also the resonant rf frequency needed to cause a nuclear spin flip. This would make it possible for a resonant rf pulse to selectively change the state of only that nucleus. Between any two neighboring A gates would be a 'J gate', for affecting the overlap between two electron orbitals bound to neighboring phosphorus nuclei in the lattice. This J gate would result in an indirect coupling between the two neighboring phosphorus qubits, making it possible to implement the quantum gates necessary for quantum computation. Since it is presently prohibitively difficult to directly measure the spin state of an individual nucleus, an indirect approach might be implemented, involving a chain of interactions among the nuclear spin, its bound electron and a neighboring electron, the external magnetic field, and the J gate overlaying the two electron orbitals; these interactions could affect the capacity between neighboring A gate electrodes, which could be measured. Normally, all electron spins would be pointed in the direction of the external magnetic field, but if the overlap between two neighboring electron orbitals were sufficiently increased by an applied J-gate voltage, it might become energetically favorable for the pair of electrons to change their state so that their spins are opposite. Whether this happens would depend on the direction of whichever phosphorus spin is coupled most strongly to its bound electron, and that would depend on the A-gate voltage. The Pauli exclusion principle would not allow both electrons to hop into the same atom unless their spins were opposite, and a hop would change the capacitance between the neighboring A electrodes. This same mechanism may also enable qubit states to be initialized, since each can be measured individually, and the measured state could be reversed with an NMR pulse if necessary. Neither controlled qubit entanglement, two-qubit gates, nor quantum logic has yet been demonstrated for the silicon-based quantum computer concept. Unless such elementary operations can be robustly demonstrated, then the scalability supposedly offered by this approach or any other will be irrelevant. Much greater effort is needed in the calculation and measurement of various possible spin decoherence mechanisms for electrons and nuclei at donors, free electrons in real silicon heterostructure materials, and entangled states. Some possible sources of qubit decoherence include solid state fluctuations, erroneous donor locations, silicon isotope contamination, phonon scattering, dipolar interactions between electron spins, electromagnetic field fluctuations, low-frequency gate voltage fluctuations, thermal fluctuations of gate voltages, charge motion within the semiconductor host, and gate calibration noise. Before such a device can be successfully implemented, many formidable technological problems must be overcome, including: single-spin measurement; demonstrating robust two-qubit operations; emplacement of individual phosphorus atoms in a prescribed regular array in a perfect silicon crystal; development of defect-free semiconductor and overlaying layers; limitation of the decoherence rate of the phosphorus qubits in

the presence of electrode fluctuations, gate biasing, rf-induced eddy currents, charge fluctuations, spin impurities, and crystal defects; sufficient limitation of the probability of error in each operation; and nanoscale fabrication. If these and other obstacles can be overcome, this approach may offer the possibility of a quantum computer which is scalable to the large number of qubits required for large scale quantum computation

Another popular potentially scalable solid-state approach to quantum computer development is the quantum-dot quantum computer. Various approaches have been considered [64,116]. In one popular concept [117–120], a qubit would be the two spin states of an electron in a single-electron quantum dot, and a quantum register would consist of an array of coupled single-electron quantum dots. Each semiconductor quantum dot would consist of one excess electron with spin 1/2 in a potential well that confines the electron in all three dimensions. Quantum gate operations would be performed by gating of the tunneling barrier between neighboring dots, to produce controlled entanglements of the qubits. The tunnel barrier between dots could be raised or lowered by the application of a higher or lower gate voltage. If the barrier were sufficiently reduced, virtual tunneling could occur, resulting in transient spin-spin coupling. Hopping to a neighboring auxiliary ferromagnetic dot might be used to implement single qubit operations. Also, readout might be implemented through tunneling to a neighboring auxiliary paramagnetic dot, which could nucleate a ferromagnetic domain that could then be measured. Alternatively, spin-dependent tunneling into another neighboring auxiliary dot might enable spin measurement by means of an electrometer. Reversing this procedure might accomplish general state preparation. Ground state preparation could be accomplished by cryogenic cooling in a uniform applied magnetic field. Quantum computer implementations have also been proposed that would use electron spin in quantum dots manipulated with cavity QED methods [121]. Some possible sources of spin-qubit decoherence include spin-orbit coupling, coupling to environmental electron and nuclear spin states, any interactions ignored in the qubit Hamiltonian, magnetic field inhomogeneities and fluctuations, off-resonant spin-flip transitions, and hyperfine interactions [63,116,122]. In other approaches based on quantum dots, the qubit does not consist of electron spin states, but instead of pseudo-spin states, corresponding to charged orbital degrees of freedom [123,124]. Pseudo-spin can have numerous internal degrees of freedom which can cause decoherence. (Real spin has the advantage of permanent well-defined qubits with no extra dimensions for qubit state leakage, and much longer dephasing times.) Gate operation may also be performed by spectroscopic manipulation. Coherent optical control of quantum-dot states is an important area of research. Picosecond optical excitation can be used to coherently control quantum-dot states on a time scale which is small compared to the decoherence time. Some encouragement for the charge-qubit approach was offered by the experimental demonstration of exciton entanglement in a single quantum dot [125,126], and also in a pair of quantum dots [127]. However, no two-qubit gate operations involving neighboring dots have yet been demonstrated in any quantum dot approach. Exciton-based qubits are many-body excitations and may not be sufficiently stable and robust to enable large scale controlled entanglement. Possible sources of charge qubit decoherence include voltage fluctuations on control gates, uncontrolled distant charge motion, phonon interactions, and other scattering mechanisms. Generally it is expected that solid-state systems, because of their complex internal field and many-particle environment, will subject qubit states to numerous possible mechanisms of quantum decoherence, presenting formidable obstacles to the development of a practical large-scale quantum computer based on the quantum-dot approach.

11 SUPERCONDUCTING QUANTUM COMPUTERS

Efforts also are under way to develop scalable superconducting quantum computers. First consider the Josephson junction quantum computer [52,62,63,99,128–131]. In one relatively simple exploratory approach, a nanoelectronic device would consist of an array of low-capacitance Josephson junctions [130,131]. The device would exploit coherent tunneling in the superconducting state, with the possibility of controlling individual charges by means of Coulomb blockade effects. The Josephson junction qubit is implemented in a small superconducting island connected by a tunnel junction to a superconducting electrode. The qubit consists of two charge states of the superconducting island adjacent to the junction. The logical states differ by one Cooper-pair charge. A Cooper pair consists of two electrons bound together through a phonon interaction. The island is connected to

an ideal voltage source with a gate capacitor between them. An array of such Josephson junction qubits, each with its own voltage source, could be connected in parallel with each other and also with a mutual inductor. The array would serve as a quantum register. One- and two-qubit gates may be implemented by application of appropriate sequences of voltages across the junctions, and by the tuning of selected qubits to resonance. Readout might be accomplished by capacitively coupling a dissipative normal-metal single-electron transistor to a qubit. This simple design presents various challenges: it requires high-precision timing control, and it involves residual two-qubit interactions that will produce errors. An improved design has been considered in which the Josephson junctions are replaced by SQUIDs (superconducting quantum interference devices), which can be controlled by magnetic fluxes [130]. This design might enable exact on-off switching of the two-qubit coupling, relaxation of the timing control and system parameter requirements, and complete control of two-qubit couplings. Parallel operations on different qubits might be achieved, in principle, by more advanced designs, including additional tunable SQUIDs to decouple different parts of the circuit. Scaling to large numbers of qubits with massively parallel operation will necessitate much more elaborate designs, significant progress in nanotechnology, reduction in working temperature, near perfect control of time-dependent gate voltages, and much longer decoherence times. Some possible sources of decoherence for charge qubits include the electromagnetic environment, spontaneous photon emission, external charge and voltage noise due to charge impurities in fabricated circuits or voltage fluctuations in the current coupled to the qubits, quasiparticle effects in the superconducting circuits, coupling to electronic measuring device and control circuits, and coupling to spin impurities in the solid state environment [63].

The quantized flux in a SQUID can also be used as the basic qubit, instead of the charge of a Josephson junction island. This would provide the basis for a potential SQUID quantum computer [62,132–135]. Although SQUIDs are macroscopic objects, and macroscopic objects generally suffer decoherence in the extreme, many of the dissipative mechanisms that normally operate in macroscopic systems can be eliminated in SQUID systems [3,136,137]. The Hamiltonian of an rf SQUID can be represented as a two-state system [137]. The SQUID consists of a single tunnel junction with critical current shunted by an inductor. If a magnetic flux of half the fundamental flux quantum is applied to the loop, and appropriate constraints on the critical current and inductance are satisfied, then a two-state system can be created, in which the two states correspond to the loop containing either one flux quantum or none at al [132]. A supercurrent then circulates the SQUID ring in either direction. Important issues in the SQUID approach to quantum computer development include the required operating temperature, required junction quality, suppression of competing modes, magnetic coupling of flux qubits to magnetic impurities, unidentified decoherence mechanisms, sufficiently small junction capacitances, and required fabrication technology. Some possible sources of decoherence for flux qubits include external flux/current noise due to magnetic impurities in the fabricated circuit or current fluctuations in the circuits coupled to the qubit, quasiparticle effects in the superconducting circuit, magnetic fluctuations due to nuclear spins in the solid, electromagnetic radiation by the qubit, and unwanted coupling to other flux qubits [62,63].

It is encouraging for the superconducting approach that coherent tunneling of Cooper pairs, Cooper-pair qubit control and Rabi oscillations, resonant tunneling of the flux between quantized energy levels in different SQUID flux states, and quantum superposition of states have been theoretically investigated and experimentally demonstrated [52,53,62,63,128,129,133–136,138–144]. One must also be able to entangle multiple qubits with each other, and there may be decoherence mechanisms that can only occur for highly entangled states. No two-qubit operations or quantum logic gates have yet been experimentally demonstrated in any superconducting approach.

12 NEUTRAL ATOM QUANTUM COMPUTERS

Another innovative approach to quantum computer development involves trapped atoms in optical lattices. Lasers can be used to confine ultracold atoms in periodic lattices. The atoms are held together with light (instead of chemical bonds, as in a solid). Laser cooling and trapping techniques, used in producing Bose-Einstein condensation, are also used to form optical lattices. In an optical lattice, ultracold atoms can be arranged in a crystal-like array in an optical potential in which the intensity or polarization of light varies periodically. Near

zero temperature, webs of interfering laser beams can be used to cool a collection of atoms, and the atoms become suspended in well-defined positions in the interfering beams. The separations of the atoms in an optical lattice are hundreds of times that in an ordinary solid. A goal is to achieve a high filling factor with all lattice sites occupied. The potential well depth is far less than that in a solid, and the dynamical oscillations of the atoms are far lower in frequency than in a solid. Defects and impurities are absent in the optical lattice. Through changes in the polarization and the direction of propagation of the laser beams, many different crystalline structures can be created in one, two, or three dimensions. The optical potential seen by an atom in an optical lattice depends on the magnetic quantum number of the atom. Dissipation and decoherence in optical lattices occurs due to spontaneous emission. Evaporative cooling may be able to produce lattices in which every site is filled. Atoms can be trapped in a two-dimensional lattice and cooled to the zero point of motion by resolved-sideband cooling. These characteristics are important for initial state preparation and the manipulation of quantum states.

An optical lattice may serve as the arena for neutral atom quantum computers. It may become possible to implement quantum logic with neutral atoms trapped in an optical lattice, very far off resonance [63,145–150]. A qubit would consist of two states of an atom. If the lasers are detuned very far off resonance, photon scattering is made negligible, and high laser intensities can maintain substantial potential wells. By means of laser cooling, the atoms can be prepared in the ground state of the potential well. The lattice geometry can be varied dynamically: changing the angle between different laser polarizations can control the distance between wells. Two atoms trapped in neighboring wells can be forced into the same well by varying the polarization of the trapping lasers. An auxiliary laser can then induce a near-resonant electric dipole, and the electric dipole-dipole potential can provide the predominant interaction between the atoms. Following this, the atoms can be separated by adiabatic rotation of the laser polarization. Quantum gates may be implemented through the induced coherent dipole interactions. Single qubit operations could be performed with polarized resonant Raman pulses. Two-qubit operations require conditioning the state of one atom on that of the other. A controlled-NOT gate could be achieved by conditioning the target atomic resonance on a resolvable level shift induced by the control atom. The resonant dipoles would be conditionally turned on only during conditional logic operations, and environmental decoherence would be thereby suppressed. Large numbers of atoms could be entangled by a sequence of two-qubit interactions. If the atoms are lightly confined to separations small relative to the wavelength, then a coherent dipole-dipole interaction could be induced with negligible photon scattering. The coherent level shift could thereby be substantially enhanced, while the cooperative emission rate could be substantially suppressed. The atoms couple very weakly to the environment and would interact only during two-qubit logical operations, and all manipulations would be performed rapidly relative to the photon scattering rate, thus impeding spontaneous emission, which is the main source of decoherence. Some possible sources of quantum decoherence in neutral atom quantum computers may include spontaneous emission, photon scattering, unwanted elastic and inelastic collisions, unwanted entanglement between motional and internal states, spin couplings, motional state perturbations, trapping-field fluctuations, time-varying magnetic fields, and laser noise. Although single-qubit rotations are readily performed, no two-qubit gates have yet been implemented in the neutal atom approach. The decoherence time must be much longer than the two-qubit gate time. Also, before an operational neutral atom quantum computer can be successfully developed to perform even elementary quantum computations, many issues must be explored, including increasing the filling fraction of atoms in the lattice, developing methods for addressing and reading out individual qubits, investigating the effects of atomic collisions, and implementing quantum error-correction methods.

Bose condensates in optical lattices may provide a further innovation in quantum computer development. Bose condensates can be confined in an optical dipole trap. They can also be created in an optical lattice, and the theory of condensates in optical potentials has been investigated. A very innovative scheme was proposed [151] to fill an optical lattice with a Bose condensate, and exploit ideas related to Mott transitions in optical lattices [152]. A far-detuned optical lattice acts as a conservative potential and can be loaded with a Bose condensed atomic vapor, resulting in tens of atoms per lattice site. It was argued [152] that the dynamics of bosonic atoms corresponds to that of a Bose-Hubbard model, which describes the hopping of bosonic atoms between the lowest vibrational states of lattice sites. The important system parameters can be controlled by appropriate laser parameters and configurations. The model predicted a phase transition from the superfluid phase to the Mott insulator phase at low temperature. This can result in the formation of an optical crystal with long range order and period controlled by the laser light. A finite gap would be produced in the excitation spectrum. The superfluid/Mott-insulator

transition was observed for the first time in recent experiments [153]. Average occupations of several atoms per well were achieved. An optical crystal could be created with uniform lattice occupation, or tailored atomic patterns could be produced. This would occur at sufficiently low temperature that cold laser-controlled coherent interactions could implement conditional dynamics in moving trap potentials. Methods have been investigated for producing two-qubit quantum gates and highly entangled states, and may provide the basis for a possible Bose-condensate quantum computer.

13 QUANTUM ERROR CORRECTION

Quantum error-correction methods may provide the means to successfully combat decoherence in quantum computers and other qubit devices [56,59,64,69,99,154–184]. Quantum error correctors are implementations of these methods that involve quantum circuits consisting of networks of quantum gates. The interaction of a qubit in a general state,

$$|\phi\rangle = a\,|0\rangle + b\,|1\rangle\,, \tag{82}$$

with its environment in state $|e\rangle$, results, in general, in the following entanglement between the qubit and its environment [69,179]:

$$|e\rangle\,|\phi\rangle = |e\rangle\,(a\,|0\rangle + b\,|1\rangle) \Rightarrow a\,(c_{00}\,|e_{00}\rangle\,|0\rangle + c_{01}\,|e_{00}\rangle\,|1\rangle) + b\,(c_{10}\,|e_{10}\rangle\,|1\rangle + c_{11}\,|e_{11}\rangle\,|0\rangle)\,, \tag{83}$$

where $|e_{ij}\rangle$ denote states of the environment, and c_{ij} are complex coefficients that depend on the environmental interactions with the qubit. Equivalently, Eq. (83) can be rewritten as follows [69]:

$$|e\rangle\,|\phi\rangle \Rightarrow (|e_I\rangle\,I + |e_X\rangle\,X + |e_Y\rangle\,Y + |e_Z\rangle\,Z)\,|\phi\rangle\,, \tag{84}$$

where the operators $\{I, X, Y, Z\}$ are defined by

$$I \equiv |0\rangle\,\langle 0| + |1\rangle\,\langle 1| = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{85}$$

$$X \equiv |0\rangle\,\langle 1| + |1\rangle\,\langle 0| = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{86}$$

$$Z \equiv |0\rangle\,\langle 0| - |1\rangle\,\langle 1| = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{87}$$

$$Y \equiv XZ = |1\rangle\,\langle 0| - |0\rangle\,\langle 1| = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{88}$$

The corresponding matrix representatives are also given in Eqs. (85) to (88) (these matrices can also be simply related to the Pauli spin matrices). Note that by completeness, I, in Eq. (85), is the identity operator, which corresponds to the unit matrix. Also, the operator X, Eq. (86), is the NOT operator, since in matrix form one has

$$X = [\langle m|X|n\rangle] = [\langle m|\,(|0\rangle\,\langle 1| + |1\rangle\,\langle 0|)\,|n\rangle] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = N. \tag{89}$$

Analogously, one obtains the matrix representatives shown for Z and Y in Eqs. (87) and (88), respectively. Also, in Eq. (84), the states $|e_I\rangle$, $|e_X\rangle$, $|e_Y\rangle$, and $|e_Z\rangle$ are given by

$$|e_I\rangle = 2^{-1}\,(c_{00}\,|e_{00}\rangle + c_{10}\,|e_{10}\rangle)\,, \tag{90}$$

$$|e_X\rangle = 2^{-1}\,(c_{01}\,|e_{01}\rangle + c_{11}\,|e_{11}\rangle)\,, \tag{91}$$

$$|e_Y\rangle = 2^{-1}\,(c_{01}\,|e_{01}\rangle - c_{11}\,|e_{11}\rangle)\,, \tag{92}$$

$$|e_Z\rangle = 2^{-1}\,(c_{00}\,|e_{00}\rangle - c_{10}\,|e_{10}\rangle)\,. \tag{93}$$

Equation (84) represents three types of errors, corresponding to the operators X, Y, Z. The operator X represents a bit flip, since it interchanges the basis states, thus:

$$X \begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix} = \begin{pmatrix} |1\rangle \\ |0\rangle \end{pmatrix}. \tag{94}$$

The operator Z represents a phase error, since it introduces a relative phase $e^{i\pi} = -1$:

$$Z \begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix} = \begin{pmatrix} |0\rangle \\ -|1\rangle \end{pmatrix}. \tag{95}$$

The operator $Y = XZ$ represents a phase change together with a bit flip, since

$$Y \begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix} = \begin{pmatrix} -|1\rangle \\ |0\rangle \end{pmatrix}. \tag{96}$$

To see that Eq. (84) is equivalent to Eq. (83), note that if we use Eqs. (82), (85)–(88), and (90)–(93), then

$$\begin{aligned} |e\rangle |\phi\rangle \Rightarrow & \left(|e_I\rangle I + |e_X\rangle X + |e_Y\rangle Y + |e_Z\rangle Z\right) |\phi\rangle \\ & = 2^{-1} \left(c_{00} |e_{00}\rangle + c_{10} |e_{10}\rangle\right) \left(|0\rangle \langle 0| + |1\rangle \langle 1|\right) \left(a |0\rangle + b |1\rangle\right) \\ & \quad + 2^{-1} \left(c_{01} |e_{01}\rangle + c_{11} |e_{11}\rangle\right) \left(|0\rangle \langle 1| + |1\rangle \langle 0|\right) \left(a |0\rangle + b |1\rangle\right) \\ & \quad + 2^{-1} \left(c_{01} |e_{01}\rangle - c_{11} |e_{11}\rangle\right) \left(|1\rangle \langle 0| - |0\rangle \langle 1|\right) \left(a |0\rangle + b |1\rangle\right) \\ & \quad + 2^{-1} \left(c_{00} |e_{00}\rangle - c_{10} |e_{10}\rangle\right) \left(|0\rangle \langle 0| - |1\rangle \langle 1|\right) \left(a |0\rangle + b |1\rangle\right) \\ & = 2^{-1} \left(c_{00} |e_{00}\rangle + c_{10} |e_{10}\rangle\right) \left(a |0\rangle + b |1\rangle\right) \\ & \quad + 2^{-1} \left(c_{01} |e_{01}\rangle + c_{11} |e_{11}\rangle\right) \left(b |0\rangle + a |1\rangle\right) \\ & \quad + 2^{-1} \left(c_{01} |e_{01}\rangle - c_{11} |e_{11}\rangle\right) \left(-b |0\rangle + a |1\rangle\right) \\ & \quad + 2^{-1} \left(c_{00} |e_{00}\rangle - c_{10} |e_{10}\rangle\right) \left(a |0\rangle - b |1\rangle\right) \\ & = a \left(c_{00} |e_{00}\rangle |0\rangle + c_{01} |e_{01}\rangle |1\rangle\right) + b \left(c_{10} |e_{10}\rangle |1\rangle + c_{11} |e_{11}\rangle |0\rangle\right). \end{aligned} \tag{97}$$

Thus Eq. (84) is equivalent to Eq. (83).

Suppose that a quantum computer manipulates k qubits in the general state $|\phi_k\rangle$. Then, add $n-k$ qubits in the state $|0\rangle$ to the computer, so that there are n qubits. Next perform the encoding operation

$$E\left(|\phi_k\rangle |0\rangle\right) = |\phi_E\rangle, \tag{98}$$

which produces some, in general, entangled state $|\phi_E\rangle$ of all n qubits. Then, let noise affect all n qubits. The noise can be represented as a sum of error operators M, where each M is a tensor product of n operators I, X, Y, Z, one acting on each qubit. For example, if I operates on qubit 1, X on 2, Z on 3, X on 4, Y on 5, X on 6, Y on 7, and I on 8, this can be represented by the operator

$$M = I_1 X_2 Z_3 X_4 Y_5 X_6 Y_7 I_8. \tag{99}$$

Then, general interactions between the n qubits and the environment produce the general noisy state

$$|\psi\rangle_N = \sum_s |e_s\rangle M_s |\phi_E\rangle, \tag{100}$$

where each M_s is an operator involving products of the four operators I, X, Y, Z, such that each of the n qubits is acted on by one of them. Next, add another $n-k$ ancilla qubits, prepared in the state $|0\rangle_a$. For any encoding E, there is some operator A, called the syndrome extraction operator, which identifies the type of corrected error, namely [69],

$$A\left(M_s |\phi_E\rangle |0\rangle_a\right) = \left(M_s |\phi_E\rangle\right) |s\rangle_a \ \forall M_s \in S, \tag{101}$$

where S is the set of correctable errors and depends on the encoding. Here, the symbol s in $|s\rangle_a$ is a binary number that identifies the error operator M_s considered, and the states $|s\rangle_a$ are mutually orthogonal. For the

simple case that the general noise state $|\psi\rangle_N$, Eq. (100), contains only $M_s \in S$, the joint state of the n non-ancilla qubits, environment, and ancilla (following the syndrome extraction) is given by

$$|\psi\rangle_{Na} = A\,|\psi\rangle_N\,|0\rangle_a = A\left[\left(\sum_s |e_s\rangle\, M_s\,|\phi_E\rangle\right)|0\rangle_a\right] = \sum_s |e_s\rangle\,(M_s\,|\phi_E\rangle)\,|s\rangle_a\,. \tag{102}$$

If the ancilla state is measured with the measurement operator $|s\rangle_{a\,a}\langle s|$, then

$$\begin{aligned}(|s\rangle_{a\,a}\langle s|)\,|\psi\rangle_{Na} = |s\rangle_{a\,a}\langle s|\sum_{s'}|e_{s'}\rangle\,(M_{s'}\,|\phi_E\rangle)\,|s'\rangle_a = |s\rangle_a\sum_{s'}|e_{s'}\rangle\,(M_{s'}\,|\phi_E\rangle)\,\delta_{ss'}\\ = |e_s\rangle\,(M_s\,|\phi_E\rangle)\,|s\rangle_a\,;\end{aligned} \tag{103}$$

that is, the entire state collapses into $|e_s\rangle\,(M_s\,|\phi_E\rangle)\,|s\rangle_a$ for a particular s. Thus, the measurement reveals the value s, thereby determining the error operator M_s (s is the error syndrome). Next, if the operator M_s^{-1} is applied to the measured state by means of various quantum gates X, Y, or Z, there results

$$M_s^{-1}\,(|s\rangle_{a\,a}\langle s|\,|\psi\rangle_{Na}) = M_s^{-1}\,[|e_s\rangle\,(M_s\,|\phi_E\rangle)\,|s\rangle_a] = |e_s\rangle\,|\phi_E\rangle\,|s\rangle_a\,, \tag{104}$$

resulting in the noise-free state $|\phi_E\rangle$. The state $|e_s\rangle$ of the environment, appearing here, is of no interest, and the ancilla state $|s\rangle_a$ can be put back in state $|0\rangle_a$ and used again. If the noise in $|\psi\rangle_N$, Eq. (100), contains errors M_s that are not in the correctable set S, then the probability must be large that when the syndrome is extracted, the state collapses onto a correctable state. The error-correction procedure must be such that the encoding operation E and the extraction operation A are such that the set S of correctable errors includes all likely errors. If uncorrelated stochastic noise is assumed, for which the effect on a qubit at different times is uncorrelated (and the effect on different qubits is uncorrelated also), then all possible error operators can be categorized in terms of their likelihood. Those affecting fewer qubits are more likely. If a quantum error-correcting code is such that all errors affecting up to t qubits are correctable, then the code is a t-error correcting code [162,163,166,168,183]. Errors can also occur in the ancilla, quantum gates, and measurements. Methods were discovered by which the error correction suppresses more noise than it produces [170,177,179,184]. Error-correcting codes may require an extremely large overhead in terms of the numbers of qubits (required for sufficient redundancy to recover from errors) and of gates (required to process the redundantly encoded data and to diagnose and reverse errors). The error probability per qubit per gate must be very small (below the accuracy threshold) if the error correction is to succeed for arbitrarily long computations [173,175]. The requirements are formidable for reliable quantum computing using such fault-tolerant quantum error-correcting codes [172,180]. Furthermore, quantum error-correcting codes usually assume that errors in distant qubits are at most weakly correlated, and the codes are inadequate to deal with strongly correlated errors involving many qubits [180].

Note that much simpler quantum error-correction methods can be used if enough is known about the sources of noise [45,181]. Several passive error-prevention schemes have been proposed, in which the encoding occurs within subspaces that do not decohere because of symmetry properties [185–189]. It has been argued [187], on the basis of a semigroup description of quantum decoherence [190,191], that error-free quantum computation is possible in decoherence-free subspaces. The evolution of the computational degrees of freedom, which form a subspace of the total Hilbert space describing the quantum dynamics of the qubit device and its environment, is nonunitary, and is described by a semigroup. The decoherence-free subspaces are spaces spanned by states annihilated by all error generators (the operators X, Y, Z in Eq. (84) are error generators). Also, various methods of decoherence control are currently under investigation. These include the application of feedback [192,193] and of external controllable interactions [194–196]. It is argued that the effects of qubit-environment interactions can be removed by suitable decoupling perturbations acting on the qubit device over time scales comparable to the correlation time of the environment.

Quantum decoherence is also an issue in quantum communication systems, in which entanglement between widely separated states is exploited. When quantum information is sent through a communication channel such as an optical fiber, the photons encoding the information interact with the channel material and become entangled with it. This results in decoherence of the information states. Quantum error correction offers one

solution, but another solution is provided by entanglement purification [59,159,183,197,198]. In entanglement purification, entangled Einstein-Podolsky-Rosen pairs of photons, whose fidelity has been degraded by quantum decoherence, are distilled to a smaller number of high-fidelity pairs by means of certain local unitary operations and measurements performed at each end of the channel, together with postselection by classical communication. Quantum repeaters based on entanglement purification may significantly increase the achievable range of quantum communication [199].

14 SUMMARY

Following a general discussion of the physics of quantum decoherence, we reviewed the formulation of the persistence probability for a qubit device, as the probability of measuring its computational degrees of freedom in the unperturbed qubit state without the decoherence arising from environmental interactions. Also, a double path integral expression was derived for the qubit persistence probability of a qubit device interacting with its internal and external environment represented by continuous fields. Examples of the calculation of qubit persistence probability were presented for a generic single-qubit device coupled to a thermal environment, and a trapped-ion quantum register coupled to its ion vibrational modes. A review was also given of various possible approaches to quantum computer implementation, along with possible sources of quantum decoherence. Also, possible methods were briefly reviewed for correcting qubit error, including quantum error correction, decoherence avoidance and control, and entanglement purification. The real feasibility of developing a robust large-scale quantum computer by any of the proposed approaches remains in question.

The detailed physics of quantum decoherence in qubit devices will be as diverse as the possible devices one might consider, together with the variety of particular dominating environmental interactions. The structure of the corresponding qubit-device Hamiltonians, together with the relevant environmental interaction terms, will result in wide variations in the physical details of the decoherence process. Although decoherence is widespread in macroscopic, mesoscopic, and many microscopic systems, it does not have a universal description. It is, therefore, important that the phenomenon of quantum decoherence be much more extensively and thoroughly investigated, both experimentally and theoretically, if useful qubit devices are to become a reality.

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