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Vladimir Kuzmiak
Peter Markoš
Tomasz Szoplik
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Introduction

Metamaterials have become an influential concept which has a profound impact on physics, optics, and engineering communities. Unique electromagnetic features of metamaterials that have been demonstrated over the past two decades have created prospects of advancement in new applications such as cloaking, superlensing, and hyperlensing. When the metamaterial features are not limited to negative refraction only, similarity of properties of metamaterials, photonic crystals, plasmonic, and metal-dielectric multilayer structures lead to entirely new limits of knowledge and unexpected applications.

This fourth conference (the second one held in Prague) in a series of SPIE conferences on metamaterials brought together the two scientific communities of metamaterials and plasmonics. It provided an overview of recent activities in the field of complex metamaterial structures, their coupling properties, hybridization of both the electric and magnetic resonances, and latest achievements in tunable and nonlinear metamaterials. Both device applications and progress in modeling metamaterials were presented. New concepts of plasmonic devices for light transport in the subwavelength scale and their use in nano-optics systems were discussed in several interesting presentations.

This conference was one of 10 conferences held at the SPIE *Europe* Congress on Optics and Optoelectronics organized in Prague by the Institute of Physics, Academy of Science of the Czech Republic and SPIE Europe. The Congress brought together leading scientists to an important Middle Europe forum addressing the most important developments in the field of photonics and optoelectronics.

As chairs of this meeting we would like to express our thanks to all those participants who contributed through their presentations and to the program committee members.

Vladimir Kuzmiak
Peter Markoš
Tomasz Szoplik

PHOTON PHYSICS: FROM WAVE MECHANICS TO QUANTUM ELECTRODYNAMICS

Ole Keller

Institute of Physics and Nanotechnology, Aalborg University, Denmark

ABSTRACT

When rewritten in an appropriate manner, the microscopic Maxwell-Lorentz equations appear as a wave-mechanical theory for photons, and their quantum physical interaction with matter. A natural extension leads from photon wave mechanics to quantum electrodynamics (QED). In its modern formulation photon wave mechanics has given us valuable new insight in subjects such as spatial photon localization, near-field photon dynamics, transverse photon mass, photon eikonal theory, photon tunneling, and rim-zone electrodynamics. The present review is based on my plenary lecture at the SPIE-Europe 2009 Optics and Optoelectronics International Symposium in Prague.

1. INTRODUCTION

The birth of quantum field radiation theory in the years 1925-30, did not cause physicists to give up the idea of a particle concept of light. Beautiful wave mechanical theories for the photon were thus established by Landau and Peierls in 1930,[1] and by Oppenheimer[2] the year after. For a brief historical review of photon wave mechanics the reader may consult reference three.[3] Over the years physicists interest in photon wave mechanics, and its extension to the field-quantized level, has waxed and waned, but never fallen to rest.[4] In the present review, I look at this almost century old field in a modern perspective, and point out some of the modern perspectives for low-energy physical optics.

It has been known for almost two centuries that the eikonal equation in geometrical optics is form-identical to the Hamilton-Jacobi equation for Hamilton's characteristic function in Newtonian mechanics.[5,6] This fact led almost exactly a century later De Broglie and Schrödinger to the wave mechanics of massive particles.[3,5,6] In this perspective, it is perhaps not surprising that classical electrodynamics, based on the microscopic Maxwell-Lorentz equations, may be looked upon as the wave-mechanical theory of the photon. Starting from the Klein-Gordon equation, the quantum-mechanical eikonal equation for a relativistic spinless particle subjected to an electromagnetic field is first established.

In free space various constructions (objects) can claim to represent a photon wave function. Here, we discuss two of the most attractive suggestions: the photon energy wave function,[2,4] and a wave function based on the transverse vector potential.[7] For the potential approach, we set up the photon Schrödinger equation in both momentum and configuration space. Afterwards, we discuss the transverse photon mass concept in diamagnetic field-matter interaction, and the photon eikonal theory.

Modern photon wave mechanics appears to be particularly important in the quantum theory of near-field electrodynamics.[8,9] In the rim zone of matter virtual photons play a significant role, but only in light-matter interactions. A (new) virtual photon type called a near-field photon is introduced and its wave mechanics analyzed. The near-field photon is accompanied by a gauge photon, and the two photons place in QED is discussed. I end the review by a brief discussion of the position operator problem for massive photons (plasmaritons), and emphasize the importance of the rim zone in treatments of the spatial localization problem for photons emitted from microscopic (or mesoscopic) sources.[10,11]

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2. GEOMETRICAL OPTICS AND NEWTONIAN MECHANICS IN THE HAMILTON-JACOBI FORMULATION

It has been known for almost two centuries that the dynamics of a system of interacting classical (Newtonian) particles can be described neatly on the basis of Hamilton's equations

$$\dot{q}_i = \frac{\partial}{\partial p_i} H\left(\{q\}, \{p\}, t\right) \tag{1}$$

$$\dot{p}_i = -\frac{\partial}{\partial q_i} H\left(\{q\}, \{p\}, t\right) \tag{2}$$

where q_i is the generalized coordinate number i, and p_i its associated canonical (conjugate) momentum. The dots stand for differentiation with respect to time. As indicated, the Hamilton function H depends in general on the full set of coordinates and momenta, and on time. By a particular canonical transformation, which generating function involves Hamilton's principal function $S(\{q\}, \{P\}, t)$, the new Hamiltonian can be required to vanish identically. This ensures that the new generalized coordinates and momenta ($\{P\}$) become *time-independent*, and can be taken as the initial values of the old phase space variables. Hamilton's principal function now must satisfy the partial differential equation (called the Hamilton-Jacobi equation)

$$H\left(\{q\}, \left\{\frac{\partial S}{\partial q}\right\}, t\right) + \frac{\partial S}{\partial t} = 0, \tag{3}$$

and the (old) generalized momenta are identified as

$$p_i = \frac{\partial}{\partial q_i} S\left(\{q\}, t\right), \tag{4}$$

leaving out the reference to $\{P\}$ from the notation. In the special but important case where H does not depend explicitly on time, a trial solution of the form

$$S\left(\{q\}, t\right) = W\left(\{q\}\right) - Et \tag{5}$$

leads to the Hamilton-Jacobi equation

$$H\left(\{q\}, \left\{\frac{\partial W}{\partial q}\right\}\right) = E \tag{6}$$

for Hamilton's characteristic function, $W\left(\{q\}\right)$. Usually, the constant E represents the (conserved) energy of the system. For a single particle in a time-independent potential, $V(\mathbf{r})$, the equation for Hamilton's characteristic function takes the following form:

$$\nabla W(\mathbf{r}) \cdot \nabla W(\mathbf{r}) = 2m\left(E - V(\mathbf{r})\right), \tag{7}$$

where m is the particle's mass, and $\mathbf{r}$ its position vector. With the identification in Eq. (4), which here on vector form gives $\mathbf{p} = \nabla W$, Eq. (7) expresses the fact that the particle's total energy is the sum of the kinetic and potential energies.

Geometrical optics deals with the phenomena for which the wavelength of light can be neglected. In this short-wavelength limit ($\lambda \to 0$) the optical laws may be formulated in the language of geometry, and the basic quantity is the so-called eikonal $S(\mathbf{r})$, which enters the various (monochromatic) electromagnetic field vectors ($\mathbf{V}$) via the ansatz $\mathbf{V}(\mathbf{r}) = \mathbf{V}_0 \exp\left(iq_0 S(\mathbf{r})\right)$, $q_0 = \omega/c_0$ being the vacuum wavenumber of light at the angular frequency ω. The fundamental equations of geometrical optics is the eikonal equation[6]

$$\nabla S(\mathbf{r}) \cdot \nabla S(\mathbf{r}) = n^2(\mathbf{r}), \tag{8}$$

where $n(\mathbf{r})$ is the refractive index, which in the lowest-order approximation is taken as a real quantity. As first noticed by Hamilton in 1827 there is a striking form-identity between the characteristic function [Eq. (7)] and the eikonal equation of geometrical optics [Eq. (8)]. Classical electrodynamics based in the macroscopic Maxwell equations (or in a somewhat broader perspective the microscopic Maxwell-Lorentz equations) is the covering theory of geometrical optics, and this fact led almost exactly one hundred years later De Broglie and Schrödinger to the covering theory of Newtonian mechanics, i.e., quantum mechanics, as might be well-known to the reader.

For what follows it is valuable briefly to consider the quantum mechanical eikonal theory for a relativistic spinless particle of rest mass m and charge q. Placed in an electromagnetic field described by the contravariant four-potential $\{A^\mu\} = (A^0 = \Phi/c_0, \mathbf{A})$ the classical relativistic Hamiltonian is

$$H = c_0\left[(mc_0)^2 + (\mathbf{p} - q\mathbf{A})^2\right]^{1/2} + q\Phi. \tag{9}$$

The Hamiltonian in Eq. (9) has no explicit time dependence, and with $\mathbf{p} = \nabla S$, the equation for Hamilton's principal function [Eq. (3)] can be written down. All solutions to this equation will also be solutions to the squared Hamilton-Jacobi equation, viz.,

$$c_0^2(\nabla S - q\mathbf{A})^2 + (mc_0^2)^2 = \left(\frac{\partial S}{\partial t} + q\Phi\right)^2. \tag{10}$$

The quantum mechanical eikonal equation for a relativistic spinless particle subjected to an electromagnetic field is obtained starting from the Klein-Gordon equation

$$c_0^2\left[(\frac{\hbar}{i}\nabla - q\mathbf{A})^2 + (mc_0)^2\right]\Psi(\mathbf{r},t) = \left(\frac{\hbar}{i}\frac{\partial}{\partial t} + q\Phi\right)^2\Psi(\mathbf{r},t). \tag{11}$$

By inserting the eikonal ansatz

$$\Psi(\mathbf{r},t) = \Psi_0 \exp\left(\frac{i}{\hbar}S(\mathbf{r},t)\right) \tag{12}$$

for the wavefunction, the quantum mechanical Hamilton-Jacobi equation takes the form

$$(\nabla S - q\mathbf{A})^2 + (mc_0)^2 - \frac{1}{c_0^2}\left(\frac{\partial S}{\partial t} + q\Phi\right)^2 = i\hbar\left(\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)S \tag{13}$$

in the Lorenz gauge. In the classical limit ($\hbar \to 0$), Eq. (13) is reduced to Eq. (10). In covariant notation, one has in general

$$(\partial_\mu S - qA_\mu)(\partial^\mu S - qA^\mu) + (mc_0)^2 = i\hbar\partial_\mu(\partial^\mu S - qA^\mu), \tag{14}$$

and with the Lorenz gauge condition, given by

$$\partial_\mu A^\mu = 0, \tag{15}$$

Eq. (14) is reduced to the form given in Eq. (13). The relativistic covariance of the eikonal equation is manifest when the equation is written in the form given in Eq. (14).

3. PHOTON WAVE MECHANICS

The analysis in section 2 suggests that the microscopic Maxwell-Lorentz equations somehow may be considered as a wave-mechanical theory of the photon. This possibility was originally investigated by Landau and Peierls[1] and Oppenheimer,[2] and over the years physicist's interest in photon wave mechanics has waxed and waned, but never fallen to rest. In free space, various "constructions" (objects) can rightly claim that they represent the wave function of the photon. What is important physically is that the observable predictions related to the photon-matter interaction will be identical for the various choices.

In the energy wave-function formalism two photon wave functions, $\mathbf{f}_\pm^{(+)}$, are defined as follows:[4]

$$\mathbf{f}_\pm^{(+)}(\mathbf{r},t) = \left(\frac{\epsilon_0}{2}\right)^{1/2}\left[\mathbf{e}_T^{(+)}(\mathbf{r},t) \pm ic_0\mathbf{b}^{(+)}(\mathbf{r},t)\right]. \tag{16}$$

The superscript (+) indicates that the wave function is build from positive-frequency components of the electromagnetic field. In the language of coherence theory the $\mathbf{f}_\pm^{(+)}$'s are analytical signals. The wave function of the antiphoton is constructed from the negative-frequency part of the field spectrum. Physically, the photon and antiphoton are identical. This can be inferred from the Maxwell equations. Small letters $\mathbf{e}_T$ and $\mathbf{b}$ have been assigned to the (analytical) electric and magnetic fields of the photon to indicate that the wave function has been properly normalized. To normalize the wave function of the in general non-monochromatic photon it is necessary to adjust the amplitude of the current-density source distribution correctly.[8] To emphasize that the electrical field in free space is a transverse vector field, a subscript T has been added to $\mathbf{e}^{(+)}$. As we shall understand very soon, $\mathbf{f}_+^{(+)}$ and $\mathbf{f}_-^{(+)}$ are wave functions composed of positive- and negative-helicity components. The photon helicity is a Lorentz-invariant property.

From the free-space Maxwell equations one can establish the following quantum mechanical wave equations for the two helicity species:

$$i\hbar\frac{\partial}{\partial t}\mathbf{f}_\pm^{(+)}(\mathbf{r},t) = \pm c_0\left(\frac{\mathbf{s}}{\hbar}\cdot\frac{\hbar}{i}\nabla\right)\mathbf{f}_\pm^{(+)}(\mathbf{r},t). \tag{17}$$

In Eq. (17) we have written the wave equation in Hamiltonian form. The vector $\mathbf{s}$ is the Cartesian spin-one operator of the photon. In a 3×3 matrix representation the Cartesian components of $\mathbf{s}$ are given by

$$(s_k)_{ij} = \frac{\hbar}{i}\epsilon_{ijk}, \tag{18}$$

where ϵ_{ijk} is the Levi-Civita tensor. Since $\hat{\mathbf{p}} = (\hbar/i)\nabla$ is the photon momentum operator in configuration space, the scalar product $\mathbf{s}\cdot\hat{\mathbf{p}}$ confirms that the wave functions $\mathbf{f}_\pm^{(+)}(\mathbf{r},t)$ describe photons composed of positive- and negative helicity components. Since Plancks constant (h) does not appear in the classical Maxwell-Lorentz equations, $\hbar$ can of course be eliminated from Eq. (17), as the reader readily may realize. The name photon energy wave function relates to the fact that the quantum mechanical mean value of the photon energy-momentum four vector operator $\{p^\mu\} = (\hbar\omega/c_0, \hbar\mathbf{q})$ is given by

$$\{P^\mu\} = \frac{1}{\hbar c_0}\sum_{s=+,-}\int_{-\infty}^{\infty}\{p^\mu\}\left|f_s^{(+)}(\mathbf{q};t)\right|^2\frac{d^3q}{(2\pi)^3 q}, \tag{19}$$

where the $f_\pm^{(+)}(\mathbf{q};t)$'s are the scalar photon wave functions in momentum space.

The vectorial and scalar wave functions are related by $\mathbf{f}_\pm^{(+)}(\mathbf{q};t) = f_\pm^{(+)}(\mathbf{q};t)\mathbf{e}_\pm(\mathbf{q}/q)$, where $\mathbf{e}_\pm$ are the helicity unit vectors of the + and − species. Relativistically, d^3q/q is the invariant volume element on the light cone.

It is often convenient to use spinor notation, and thus combine the two wave functions in Eq. (16) into a single one, viz.,

$$\boldsymbol{\Phi}(\mathbf{r},t) = \begin{pmatrix}\mathbf{f}_+^{(+)}(\mathbf{r},t)\\ \mathbf{f}_-^{(+)}(\mathbf{r},t)\end{pmatrix}. \tag{20}$$

The photon wave function in Eq. (20) is a six-component object.

In the present context it is useful to base photon wave mechanics on the electromagnetic potentials. In free space only a transverse vector potential exists, and this potential is gauge invariant. The analytical part of the transverse vector potential $\mathbf{a}_T^{(+)}(\mathbf{r},t)$ satisfies the wave equation

$$\Box\mathbf{a}_T^{(+)}(\mathbf{r},t) = \mathbf{0}, \tag{21}$$

where $\Box = \nabla^2 - c_0^{-2}\partial^2/\partial t^2$ is the d'Lambertian operator. Formally, one may factorize the $\Box$-operator as follows:

$$\Box = \left(\frac{i}{c_0}\frac{\partial}{\partial t} + \sqrt{-\nabla^2}\right)\left(\frac{i}{c_0}\frac{\partial}{\partial t} - \sqrt{-\nabla^2}\right), \tag{22}$$

and from this it appears that all solutions to

$$i\hbar\frac{\partial}{\partial t}\mathbf{a}_T^{(+)}(\mathbf{r},t) = \hbar c_0\sqrt{-\nabla^2}\,\mathbf{a}_T^{(+)}(\mathbf{r},t) \tag{23}$$

are also solutions to Eq. (21). One may consider $\mathbf{a}_T^{(+)}(\mathbf{r},t)$ [small letter for normalization] as the vectorial wave function for the photon in free space, and Eq. (23) as its quantum mechanical wave equation, written in Hamiltonian form. The quantity $\sqrt{-\nabla^2}$ is a spatially nonlocal operator which action in Fourier (wave-vector) space is given via

$$\sqrt{-\nabla^2}\mathbf{a}_T^{(+)}(\mathbf{r},t) = \int_{-\infty}^{\infty} q\mathbf{a}_T^{(+)}(\mathbf{q};t)e^{i\mathbf{q}\cdot\mathbf{r}}\frac{d^3q}{(2\pi)^3}. \tag{24}$$

The operator $i\sqrt{-\nabla^2}$ was originally introduced by Landau and Peierls in 1930 in connection with their attempt to establish a wave equation for the light quantum (photon). In **q**-space, Eq. (23) takes the form

$$i\hbar\frac{\partial}{\partial t}\mathbf{a}_T^{(+)}(\mathbf{q};t) = c_0\hbar q\mathbf{a}_T^{(+)}(\mathbf{q};t), \tag{25}$$

and this equation may be considered as the vectorial wave equation for a transverse photon with momentum $\mathbf{p} = \hbar\mathbf{q}$ and energy $H = pc_0$. Two transverse scalar photon wave functions can be introduced in the $(\mathbf{q};t)$-domain by expanding $\mathbf{a}_T^{(+)}(\mathbf{q};t)$ after two orthogonal (possibly complex) unit vectors (helicity unit vectors, e.g) located in a plane perpendicular to **q**.

4. TRANSVERSE PHOTON MASS. PHOTON EIKONAL THEORY

Let us now consider the photon propagation in a solid-state plasma (jellium), and let us limit ourselves to high frequencies and linear photon-matter interactions. At high frequencies the particle properties of the photon dominates over its wave properties, and the field-matter coupling is diamagnetic. In the space-frequency domain the analytical part of the transverse vectorial potential hence satisfies the integro-differential equation[12]

$$(\nabla^2 + q_0^2)\mathbf{a}_T^{(+)}(\mathbf{r};\omega) = \frac{\mu_0 e^2}{m}\int_{-\infty}^{\infty} \overleftrightarrow{\delta}_T(\mathbf{r}-\mathbf{r}')\cdot\left[N(\mathbf{r}')\mathbf{a}_T^{(+)}(\mathbf{r}';\omega)\right]d^3r', \tag{26}$$

where $N(\mathbf{r})$ is the many-body electron density, $\overleftrightarrow{\delta}_T(\mathbf{r}-\mathbf{r}')$ is the transverse delta function, $q_0 = \omega/c_0$ is the vacuum wave number of light, and e and m the electron charge and mass. In the present context there is a particularly interesting special case of Eq. (26). Thus, if the electron density variations in space are negligible $[N(\mathbf{r}) \cong N_0]$ it can be shown that $\mathbf{a}_T^{(+)}(\mathbf{r},t)$ satisfies the equation

$$i\hbar\frac{\partial}{\partial t}\mathbf{a}_T^{(+)}(\mathbf{r},t) = c_0\hbar\sqrt{Q_c^2-\nabla^2}\mathbf{a}_T^{(+)}(\mathbf{r},t), \tag{27}$$

where the operator $(Q_c^2-\nabla^2)^{1/2}$ acts as follows in Fourier space:

$$\sqrt{Q_c^2-\nabla^2}\mathbf{a}_T^{(+)}(\mathbf{r},t) = \int_{-\infty}^{\infty}\sqrt{Q_c^2+q^2}\mathbf{a}_T^{(+)}(\mathbf{q};t)e^{i\mathbf{q}\cdot\mathbf{r}}\frac{d^3q}{(2\pi)^3}. \tag{28}$$

The quantity

$$Q_c = \frac{1}{c_0}\left(\frac{N_0e^2}{m\epsilon_0}\right)^{1/2} \tag{29}$$

is the electron system's plasma wave number. One may consider Eq. (27) as the quantum mechanical wave equation for the transverse photon (plasmariton). In the wave-vector-time domain Eq. (27) takes the form

$$i\hbar\frac{\partial}{\partial t}\mathbf{a}_T^{(+)}(\mathbf{q};t) = c_0\hbar\left(q^2+Q_c^2\right)^{1/2}\mathbf{a}_T^{(+)}(\mathbf{q};t), \tag{30}$$

and from this one may show that the energy-momentum relation for the plasmariton takes the relativistic form

$$E = +\left[(pc_0)^2 + (Mc_0^2)^2\right]^{1/2}. \tag{31}$$

In a sense one may now claim that the transverse photon has acquired a finite mass

$$M = \frac{\hbar Q_c}{c_0} \tag{32}$$

in its high-frequency interaction with the jellium. The quantity $Q_c = Mc_0/\hbar$ is the Compton wave number of the massive transverse photon.

The wave equation for the plasmariton is form-identical to the Klein-Gordon equation, and the eikonal ansatz $\mathbf{a}_T^{(+)}(\mathbf{r},t) = \mathbf{a}_{T,0}^{(+)} \exp(iS(\mathbf{r},t)/\hbar)$ therefore leads to the following eikonal equation for the "free" massive photon:

$$(\partial_\mu S)(\partial^\mu S) + (Mc_0)^2 = i\hbar \partial_\mu \partial^\mu S. \tag{33}$$

If the spatial variations in $N(\mathbf{r})$ is taken into account the eikonal problem becomes more complicated.

5. THE NEAR-FIELD PHOTON CONCEPT

In near-field electrodynamics one must include two other types of photons, viz., longitudinal and scalar photons. These so-called virtual photons, which do not exist as free particles, we hence characterize by the analytical vector potentials $a_L^{(+)}(\mathbf{q};t)[\mathbf{a}_L^{(+)} = a_L^{(+)}\mathbf{q}/q]$ and $a_0^{(+)}(\mathbf{q};t)$. Instead of using the longitudinal and scalar photon variables it is convenient in near-field optics to introduce two new photon types via the unitary transformation[9]

$$\begin{pmatrix} a_{NF}^{(+)}(\mathbf{q};t) \\ a_G^{(+)}(\mathbf{q};t) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & -i \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a_L^{(+)}(\mathbf{q};t) \\ a_0^{(+)}(\mathbf{q};t) \end{pmatrix}. \tag{34}$$

The quantities $a_{NF}^{(+)}(\mathbf{q};t)$ and $a_G^{(+)}(\mathbf{q};t)$ are the quantum mechanical wave functions of the near-field (NF) and gauge (G) photons. The theory for $a_L^{(+)}$ and $a_0^{(+)}$, or equivalently $a_{NF}^{(+)}$ and $a_G^{(+)}$, always is formulated in the Lorenz gauge. The primary quantity in near-field electrodynamics is the analytical longitudinal electric field $\mathbf{E}_L^{(+)}(\mathbf{q};t) = E_L^{(+)}(\mathbf{q};t)\mathbf{q}/q$. It may be shown that the Lorenz gauge condition implies that $E_L^{(+)}(\mathbf{q};t)$ can be expressed solely in terms of either $a_{NF}^{(+)}(\mathbf{q};t)$ or $a_G^{(+)}(\mathbf{q};t)$, i.e.,

$$E_L^{(+)}(\mathbf{q};t) = \sqrt{2}\left(c_0 q + i\frac{\partial}{\partial t}\right) a_{NF}^{(+)}(\mathbf{q};t) = -i\sqrt{2}\left(c_0 q - i\frac{\partial}{\partial t}\right) a_G^{(+)}(\mathbf{q};t). \tag{35}$$

In free space the near-field and gauge photon wave functions satisfy the Hamiltonian wave equations

$$i\hbar\frac{\partial}{\partial t} a_{NF}^{(+)}(\mathbf{q};t) = c_0 \hbar q a_{NF}^{(+)}(\mathbf{q};t), \tag{36}$$

$$i\hbar\frac{\partial}{\partial t} a_G^{(+)}(\mathbf{q};t) = c_0 \hbar q a_G^{(+)}(\mathbf{q};t), \tag{37}$$

and since $E_L(\mathbf{q};t) = 0$ in free space, Eqs. (35) and (36) imply that $a_{NF}^{(+)}(\mathbf{q};t) = 0$. One cannot conclude from these equations that the wave function of the gauge photon vanishes in free space. A gauge transformation to a new (NEW) gauge within the Lorenz gauge can make $a_G^{(+)NEW}(\mathbf{q};t) = 0$. The name gauge photon originates in this property. In the presence of matter, the near-field and gauge photon variables satisfy the dynamical equations

$$\left(c_0 q + i\frac{\partial}{\partial t}\right) a_{NF}^{(+)}(\mathbf{q};t) = \frac{1}{i\sqrt{2}\epsilon_0}\frac{1}{qc_0} J_0^{(+)}(\mathbf{q};t), \tag{38}$$

$$\left(c_0 q - i\frac{\partial}{\partial t}\right) a_G^{(+)}(\mathbf{q};t) = \frac{1}{\sqrt{2}\epsilon_0}\frac{1}{qc_0} J_0^{(+)}(\mathbf{q};t), \tag{39}$$

where $J_0^{(+)}(\mathbf{q};t) = c_0\rho(\mathbf{q};t)$ is the $\mu = 0$ component of the contravariant current density four-vector.

6. RIM ZONE ELECTRODYNAMICS

The near-field photon concept connects to optical near-field interactions via Eq. (35) and (38). I shall not here undertake a systematic analysis of this connection. The longitudinal part of the electric field, $\mathbf{E}_L$, plays an important role in near-field optics, and this field relates to the near-field photon variable via the analytical relation

$$\mathbf{E}_L^{(+)}(\mathbf{r},t) = c_0\sqrt{2}\left(\frac{i}{c_0}\frac{\partial}{\partial t} + \sqrt{-\nabla^2}\right)\mathbf{a}_{NF}^{(+)}(\mathbf{r},t), \tag{40}$$

where $\mathbf{a}_{NF}^{(+)}(\mathbf{r},t)$ is the spatial Fourier transform of the vectorial quantity $a_{NF}^{(+)}(\mathbf{q};t)\mathbf{q}/q$. As an immediate consequence of Eqs. (35) and (38) the reader may obtain the following relation between the longitudinal electric field and the longitudinal source current density ($\mathbf{J}_L$):

$$\mathbf{J}_L(\mathbf{r};\omega) = i\epsilon_0\omega\mathbf{E}_L(\mathbf{r};\omega). \tag{41}$$

In the presence of a transverse current density source, $\mathbf{J}_T^{(+)}(\mathbf{r},t)$, the transverse photon variable satisfies the inhomogeneous wave equation

$$\Box\mathbf{a}_T^{(+)}(\mathbf{r},t) = -\mu_0\mathbf{J}_T^{(+)}(\mathbf{r},t), \tag{42}$$

and from this one can obtain the integral relation

$$\mathbf{E}_T(\mathbf{r};\omega) = i\mu_0\omega\int_{-\infty}^{\infty} g(|\mathbf{r}-\mathbf{r}'|;\omega)\mathbf{J}_T(\mathbf{r}';\omega)d^3r' \tag{43}$$

between the transverse parts of the electric field and source current density. The quantity $g(R;\omega) = (4\pi R)^{-1} \times \exp(iq_0R)$ is the outgoing scalar propagator. The relations in Eqs. (41) and (43) form the standard starting point for classical near-field electrodynamics. Everywhere in space outside matter one has $\mathbf{J}_L = -\mathbf{J}_T$, since $\mathbf{J} = \mathbf{0}$. It appears from the equation

$$\mathbf{J}_L(\mathbf{r};\omega) = \frac{1}{3}\mathbf{J}(\mathbf{r};\omega) + \frac{1}{4\pi}PV\int_{-\infty}^{\infty} R^{-3}\left(\overleftrightarrow{\mathbf{U}} - 3\frac{\mathbf{RR}}{R^2}\right)\cdot\mathbf{J}(\mathbf{r}';\omega)d^3r', \tag{44}$$

where $\mathbf{R} = \mathbf{r} - \mathbf{r}'$, and PV stands for principal value that $\mathbf{J}_L$ is appreciably different from zero only in a narrow zone outside a matter-filled region. This zone I have called the rim zone, and in this zone the longitudinal electric field is nonvanishing, and satisfies

$$\nabla\cdot\mathbf{E}_L(\mathbf{r};\omega) = \nabla\times\mathbf{E}_L(\mathbf{r};\omega) = 0, \tag{45}$$

the last equation per definition, of course. In the language of photon wave mechanics near-field optics describes how transverse photons via their interaction with near-field photons are generated and destroyed in space-time.

7. NEAR-FIELD AND GAUGE PHOTONS IN QED

The wave mechanical formalism can be extended to the quantum-electrodynamic level by the standard covariant quantization with an indefinite metric (Gupta-Bleuler formalism),[13].[14] The Hamilton operator of the field becomes (in box quantization)

$$\hat{H} = \sum_{\mathbf{q},r}\hbar\omega_q\hat{a}_r^\dagger(\mathbf{q})\hat{a}_r(\mathbf{q}), \tag{46}$$

where $\omega_q = c_0q$. The r-summation in Eq. (46) is over the two transverse modes (T1, T2), and the longitudinal (L) and scalar (0) modes. The annihilation ($\hat{a}_r(\mathbf{q})$) and creation ($\hat{a}_r^\dagger(\mathbf{q})$) operators of the various modes satisfy the commutation relations

$$\left[\hat{a}_r(\mathbf{q}),\hat{a}_s^\dagger(\mathbf{q}')\right] = \delta_{rs}\delta_{\mathbf{q},\mathbf{q}'}. \tag{47}$$

By extension of Eq. (34) to the operator level it is easy to show that

$$\hat{a}_L^\dagger\hat{a}_L + \hat{a}_0^\dagger\hat{a}_0 = \hat{a}_{NF}^\dagger\hat{a}_{NF} + \hat{a}_G^\dagger\hat{a}_G, \tag{48}$$

a relation which allows one to eliminate the longitudinal and scalar photon operators in the field Hamiltonian in favour of the near-field and gauge photon operators. The annihilation and creation operators of these virtual photon types satisfy the commutator relations

$$\left[\hat{a}_{NF}(\mathbf{q}), \hat{a}^{\dagger}_{NF}(\mathbf{q}')\right] = \delta_{\mathbf{q},\mathbf{q}'}, \tag{49}$$

$$\left[\hat{a}_{G}(\mathbf{q}), \hat{a}^{\dagger}_{G}(\mathbf{q}')\right] = \delta_{\mathbf{q},\mathbf{q}'}, \tag{50}$$

and the NF-G commutators vanish. In free space the quantum Lorenz condition leads for all $\mathbf{q}$ to

$$\left[\hat{a}_L(\mathbf{q}) - \hat{a}_0(\mathbf{q})\right]|\psi\rangle = \hat{a}_{NF}(\mathbf{q})|\psi\rangle = 0, \tag{51}$$

so that no physical states $|\psi\rangle$ have near-field photons. The Lorenz condition puts no restrictions on the number of gauge photons in free space. Two different gauges within the Lorenz gauge correspond to two different excitations of G-modes, but as in the classical theory the gauge photon plays no physical role in free space.

8. PLASMARITON POSITION OPERATOR. SPATIAL PHOTON LOCALIZATION

Let us now ask the following question: to what extend can a point particle in the form of a transverse (massive) photon be localized in space? If a particle's position is to be measured in quantum mechanics, the various positions in space, $\mathbf{r}_0$, should be the eigenvalues of some position operator (observable), $\hat{\mathbf{r}}$. To pin down elements of the problem, we first consider a massive relativistic particle of zero spin. For such a particle the Lorentz-invariant scalar product is given by[15]

$$\langle\psi|\phi\rangle = \int_{-\infty}^{\infty} \psi^*(\mathbf{p})\phi(\mathbf{p})\frac{d^3p}{[p^2 + (Mc_0)^2]^{1/2}} \tag{52}$$

in the momentum representation. In the plasmariton case the particle mass (M) is given by Eq. (32). Within the framework of the relativistic scalar product the Hermitian operator

$$\hat{\mathbf{r}} = i\hbar\left\{\nabla_{\mathbf{p}} - \frac{\mathbf{p}}{2[p^2 + (Mc_0)^2]}\right\} \tag{53}$$

turns out to be an acceptable position operator for a Klein-Gordon particle. In the non-relativistic (NR) limit one regains the naive position operator $\hat{\mathbf{r}}_{NR} = i\hbar\nabla_{\mathbf{p}}$. The operator in Eq. (53) satisfies the basic dyadic commutator relation

$$[\hat{\mathbf{r}}, \hat{\mathbf{p}}] = i\hbar\hat{\mathbf{1}} \tag{54}$$

where $\hat{\mathbf{1}}$ is the unit matrix operator. In the momentum representation, where the eigenvalue equation reads

$$\hat{\mathbf{r}}(\mathbf{p})\psi_{\mathbf{r}_0}(\mathbf{p}) = \mathbf{r}_0\psi_{\mathbf{r}_0}(\mathbf{p}), \tag{55}$$

the (unnormalizable) eigenfunction belonging to the eigenvalue $\mathbf{r}_0$ is given by

$$\psi_{\mathbf{r}_0}(\mathbf{p}) = \left[p^2 + (Mc_0)^2\right]^{1/4}\exp\left(-\frac{i}{\hbar}\mathbf{r}_0\cdot\mathbf{p}\right), \tag{56}$$

apart from an arbitrary multiplicative factor. In *configuration space* the eigenfunction takes the form

$$\psi_{\mathbf{r}_0}(\mathbf{r}) = 2(2\pi\hbar)^{-2}\frac{(Mc_0)^{3/2}}{|\mathbf{r}-\mathbf{r}_0|}F(Q_c|\mathbf{r}-\mathbf{r}_0|), \tag{57}$$

where

$$F(Q_c|\mathbf{r}-\mathbf{r}_0|) = \int_0^{\infty} x(1+x^2)^{1/4}\sin(Q_c|\mathbf{r}-\mathbf{r}_0|x)dx. \tag{58}$$

It appears that the function F only depends on the ratio between $|\mathbf{r}-\mathbf{r}_0|$ and the Compton wavelength of the plasmariton, $\lambda_c = h/(Mc_0)$. Since $F(z) \approx z^{-5/4}\exp(-z)$ for $z >> 1$, the wave function of a scalar particle definitely localized at $\mathbf{r}_0$ is smeared out in configuration space over a region of linear extension as the Compton wavelength around the point of localization. In the massless limit ($M \to 0$), the wave function exhibits a power-law dependence $|\mathbf{r} - \mathbf{r}_0|^{-7/2}$ around the point of localization.

So far, we have ignored the vector character of the problem. For a transverse plasmariton we may try a three-vector momentum space wave function of the type

$$\psi_{\mathbf{r}_0}(\mathbf{p}; s) = \left[p^2 + (Mc_0)^2\right]^{1/4} \mathbf{e}_{\mathbf{p}s} \exp(-\frac{i}{\hbar}\mathbf{r}_0 \cdot \mathbf{p}), \tag{59}$$

where $\mathbf{e}_{\mathbf{p}s}$ ($s = 1$ or 2) is one of two transverse unit vectors. The longitudinal unit vector, which points in the $\mathbf{p}$-direction, is denoted by $\mathbf{e}_{\mathbf{p}3}$ below. When the gradient operator, $\nabla_{\mathbf{p}}$, acts on the wave function, it mixes in a longitudinal component, and this effect in itself makes it problematic (impossible) to take the $\hat{\mathbf{r}}$-operator in Eq. (53) as the plasmariton position operator. Instead, it is perhaps natural to use the dyadic quantity

$$\hat{\mathbf{r}}_{ij} = i\hbar \left\{ \delta_{ij} \left(\nabla_{\mathbf{p}} - \frac{\mathbf{p}}{2[p^2 + (Mc_0)^2]} \right) - \sum_{s=1}^{3} \left[\nabla_{\mathbf{p}}(\mathbf{e}_{\mathbf{p}s})_i\right] (\mathbf{e}_{\mathbf{p}s})_j \right\} \tag{60}$$

as a candidate for a plasmariton position operator.[7] This choice leads to problems in configuration space, however, because the related wave function (Fourier integral transform of Eq. (59)) is not convergent. If one tries to use the standard regularization procedure the divergence-free demand on the wave function cannot be maintained.

The photon position operator problem cannot be detached from the spatial localization issue of photons emitted from a source (microscopic or mesoscopic). In the field-quantized description the relation between the transverse electric field operator, $\hat{\mathbf{E}}_T$, and the transverse current density operator, $\hat{\mathbf{J}}_T$, can be written as follows:

$$\hat{\mathbf{E}}_T(\mathbf{r}, t) = \mu_0 \int_{-\infty}^{\infty} g(|\mathbf{r} - \mathbf{r}'|, t - t') \cdot \frac{\partial}{\partial t'} \hat{\mathbf{J}}_T(\mathbf{r}', t') d^3r' dt'. \tag{61}$$

If one now replaces $\hat{\mathbf{J}}_T$ by $\hat{\mathbf{J}}$ with the help of the transverse delta function the "propagator" relation

$$\hat{\mathbf{E}}_T(\mathbf{r}, t) = \mu_0 \int_{-\infty}^{\infty} \overleftrightarrow{\mathbf{D}}_T(\mathbf{r} - \mathbf{r}', t - t') \cdot \frac{\partial}{\partial t'} \hat{\mathbf{J}}(\mathbf{r}', t') d^3r' dt' \tag{62}$$

emerges. Here, the transverse propagator has the explicit dyadic form[8]

$$\overleftrightarrow{\mathbf{D}}_T(\mathbf{R}, \tau) = -\frac{1}{4\pi R} \delta \left(\frac{R}{c_0} - \tau \right) (\overleftrightarrow{\mathbf{U}} - R^{-2}\mathbf{R}\mathbf{R}) + \frac{c_0^2 \tau}{4\pi R^3} \Theta(\tau) \Theta \left(\frac{R}{c_0} - \tau \right) (\overleftrightarrow{\mathbf{U}} - 3R^{-2}\mathbf{R}\mathbf{R}), \tag{63}$$

where Θ is the Heaviside step function. The form in Eq. (63) tells us that the transverse photon localization (in general) is limited to the extension of the rim zone, and that one cannot maintain the Einstein causality when compressing the source domain artificially from $\hat{\mathbf{J}}_T$ to $\hat{\mathbf{J}}$. Eq. (63) has served as a starting point for microscopic photon tunneling analyses in near-field electrodynamics.[16]

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