2.1.2 Reflectance, transmittance, and absorption

Reflectance ρ is the amount of flux reflected by a surface, normalized by the amount of flux incident on it. Transmittance τ is the amount of flux transmitted by a surface, normalized by the amount of flux incident on it. Any flux not reflected or transmitted is absorbed (α). Conservation of energy requires that

$$\rho + \tau + \alpha = 1. \tag{2.5}$$

By Kirchoff's radiation law,[22] the flux emitted by a hot object must be equal to the amount absorbed by it; therefore, the emittance ε of an object must be equal to α.

2.1.3 Solid angle and projected solid angle

In the spherical coordinate system shown in Fig. 2.3, the solid angle ω of an object as viewed from a particular point in space is equal to

$$\omega = \int_{\phi_1}^{\phi_2}\int_{\theta_1}^{\theta_2} \sin(\theta)\, d\theta\, d\phi, \tag{2.6}$$

where ϕ_1 and ϕ_2 define the extent of the object in the azimuthal coordinate, and θ_1 and θ_2 define the extent of the object in the elevation coordinate. The units of solid angle are steradians (sr).

A geometry often encountered in radiometry, called a *right circular cone*, is shown in Fig. 2.4, in which $\theta_1 = 0$, $\phi_1 = 0$, and $\phi_2 = 2\pi$. Its solid angle is equal to

$$\omega = 2\pi\,[1 - \cos(\theta_2)]\,. \tag{2.7}$$

Using $\theta_2 = 90$ deg in Eq. (2.7) gives the solid angle of hemisphere (2π).

The definition of projected solid angle is similar to the definition of solid angle, except for the addition of a cosine term:

$$\Omega = \int_{\phi_1}^{\phi_2}\int_{\theta_1}^{\theta_2} \sin(\theta)\cos(\theta)\, d\theta\, d\phi, \tag{2.8}$$

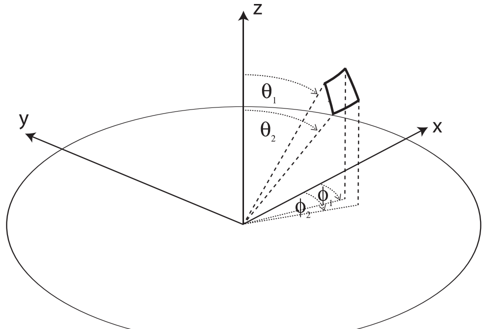

Figure 2.3 Solid angle geometry.

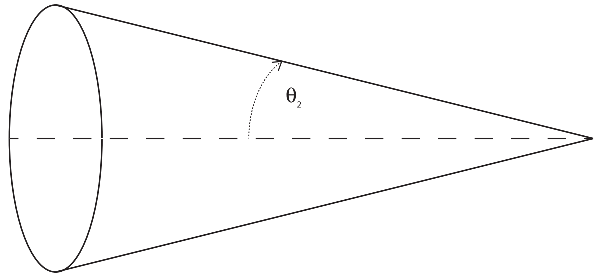

Figure 2.4 Solid angle of a right circular cone.

This geometry is illustrated in Fig. 2.5. The units of projected solid angle are steradians, just as for solid angle.

There are a number of common cases for which the value of the projected solid angle is simple to compute. The first of these is the right circular cone (shown in Fig. 2.4), which is equal to

$$\Omega = \pi \sin^2(\theta_2)\,. \tag{2.9}$$

Using $\theta_2 = 90$ deg in Eq. (2.9) gives the solid angle of a hemisphere (π). The projected solid angle divided by π is often called the *geometric configuration factor* (GCF).

The projected solid angle of an optical system can be computed from its working f-number ($f/\#'$), which is equal to

$$f/\#' = \frac{f}{D_{EP}}(1+m), \tag{2.10}$$

where f is the effective focal length of the system, D_{EP} is equal to the diameter of the entrance pupil (computed as twice the height of the marginal ray), and m is the magnification of the system (which is equal to the image distance divided by the object distance). The projected solid angle of an optical system is given by[7]

$$\Omega_{f/\#} = \frac{\pi}{4\left(f/\#'\right)^2}\,. \tag{2.11}$$

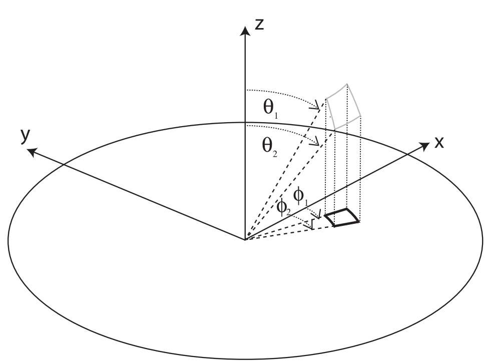

Figure 2.5 Projected solid angle geometry.

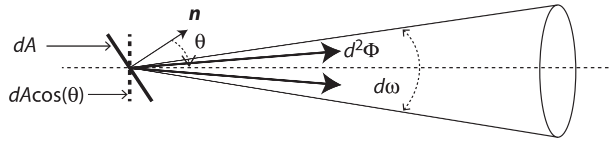

Figure 2.6 Quantities used in the definition of radiance.

2.1.4 Radiance

The radiance of a source L is equal to

$$L = \frac{d^2\Phi}{dA\cos(\theta)\,d\omega}, \tag{2.12}$$

where $d\Phi$ is the differential power emitted by the differential projected area of the source $dA\cos(\theta)$ into the differential solid angle $d\omega$, as shown in Fig. 2.6.

The units are ph/s-unit area/sr, or in photometric units as candela/m^2 (also called "nits"). Radiance is used to quantify the amount of light or "brightness" of a surface: the more flux a surface emits per unit area or the more flux it emits per projected solid angle, the greater its radiance. It is an elemental radiometric quantity, and other quantities, such as intensity or exitance (discussed later), are derived by integrating it over solid angle or area, respectively. If absorption losses are neglected, radiance is conserved through an optical system, and thus the radiance of an image is the same as the radiance of the exit pupil and of the scene (it is said to be "invariant"). A surface whose radiance is constant with respect to the emittance angle θ is said to be *Lambertian*. Though treating a surface as Lambertian is often a useful approximation, in practice no surface is perfectly Lambertian.

2.1.5 Blackbody radiance

The Planck blackbody equation can be used to compute the spectral radiance L_λ (in ph/s-cm^2-sr-μm) of an extended source from its temperature:

$$L_\lambda(\lambda, T) = \frac{C_1}{\lambda^4\left[\exp\left(C_2/\lambda T\right) - 1\right]}, \tag{2.13}$$

where $C_1 = 5.99584 \times 10^{22}$ photons-μm^5/s-cm^2-sr, $C_2 = 14387.9$ μm-K, and T is the temperature of the source in kelvin. This function is plotted in Fig. 2.7 as a function of wavelength for several temperatures.

Equation (2.13) does not account for variations in radiance versus wavelength due to changes in emissivity of the extended source. These variations, which are determined by the chemical composition of the source and which all sources have, can result in an error in the radiance predicted by the Planck equation, and therefore may need to be considered in the calculation. For example, the spectral radiance of the sun is shown in Fig. 2.8 along with the radiance predicted by the Planck equation for an ideal blackbody at 5800 K. Chemical species in the sun result in absorption bands that are not predicted by the Planck equation.

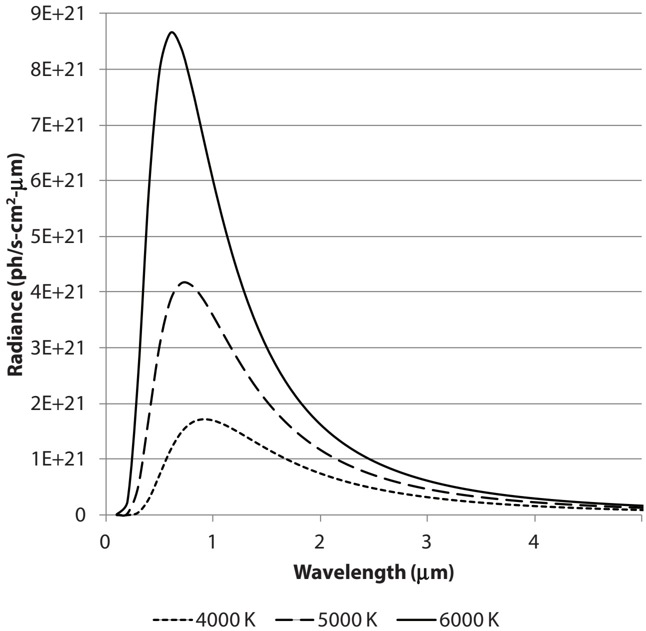

Figure 2.7 Blackbody radiance versus wavelength.

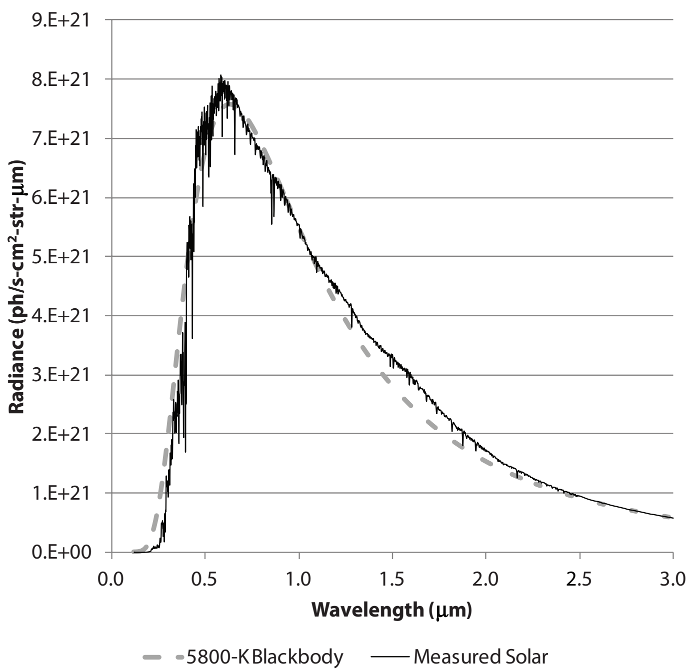

Figure 2.8 The spectral radiance of an ideal 5800-K blackbody and the measured exo-atmospheric spectral radiance of the sun.[8]

Table 2.2 Equivalent solar blackbody temperatures for typical sensor wavebands.

Waveband			Equivalent Solar BB Temperature (K)	Error in Band-Integrated Radiance
Min $\lambda(\mu m)$	Max $\lambda(\mu m)$	Name		
0.4	0.7	VIS	5848	0.436%
0.7	1	MIR	5761	−0.109%
1	3	SWIR	5986	−2.333%
3	5	MWIR	5656	1.448%
8	12	LWIR	4983	0.758%

One of the ways to reduce the magnitude of the error resulting from the use of the Planck equation is to determine the blackbody temperature of the source that minimizes the chi-squared difference between the actual spectrum and the blackbody spectrum in the waveband of interest. This was done for the spectrum shown in Fig. 2.8 (i.e., 5800 K is best-fit), and in Table 2.2 for the visible and IR sensor wavebands. The resulting error in the band-integrated radiance is also given in Table 2.2. Error analysis must be performed to determine the error in any quantity dependent on this radiance.

Figure 2.9 shows the apparent exo-atmospheric radiance of the sun, and as such it does not account for the reduction in apparent radiance due to atmospheric

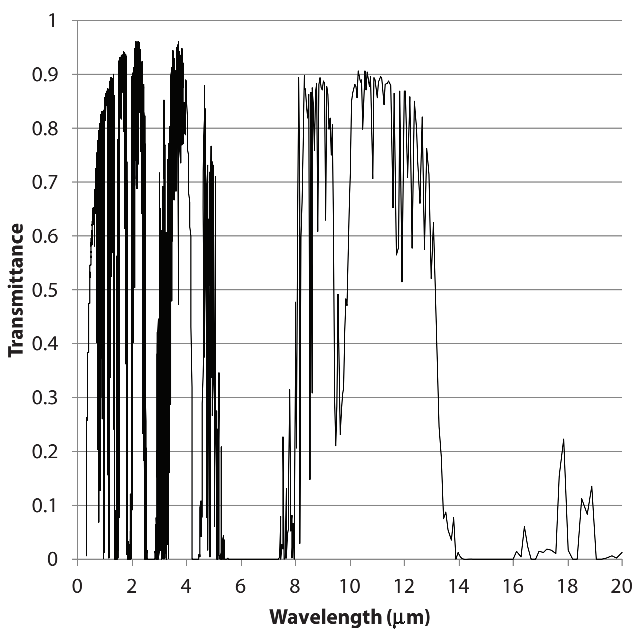

Figure 2.9 Transmittance from ground to space versus wavelength of the atmosphere, looking straight up from the ground.

Table 2.3 Band-averaged transmittance from ground to space of the atmosphere, looking straight up from the ground.

Waveband			Average Transmittance
Min λ(μm)	Max λ(μm)	Name	
0.4	0.7	VIS	0.598
0.7	1	NIR	0.722
1	3	SWIR	0.581
3	5	MWIR	0.490
8	12	LWIR	0.726

absorption and scattering (extinction), which can significantly reduce it. Atmospheric extinction is a complicated phenomenon that is a function of many variables, including wavelength, the geometry of the path of light through the atmosphere, and weather conditions, to name a few. The software MODTRAN[9] was created to deal with this complexity—it can predict atmospheric extinction as a function of these variables and more. Figure 2.9 shows the transmittance from ground to space of the default MODTRAN atmospheric model, looking straight up from the ground, which is the maximum transmittance possible from the ground to space. Table 2.3 gives the same transmittance averaged over typical sensor wavebands; this table quantifies the error in apparent radiance of the sun that results from neglecting atmospheric extinction.

The total radiance L in a waveband can be calculated by integrating Eq. (2.13) over the waveband of interest:

$$L = \int_{\lambda_1}^{\lambda_2} L_\lambda d\lambda, \tag{2.14}$$

where λ_1 and λ_2 are the minimum and maximum wavelengths of the waveband.

The wavelength λ_{peak} corresponding to the peak radiance can be computed from the temperature of the blackbody T using Wien's displacement law for photons:

$$\lambda_{peak} = \frac{3670}{T}, \tag{2.15}$$

where λ_{peak} is in μm, and T is in kelvin. Integrating Eq. (2.13) over all wavelengths gives the total radiance emitted by a blackbody L_{total}, which can be expressed using the Stefan–Boltzman law for photons:

$$L_{total} = \left(\frac{\sigma_p}{\pi}\right) T^3, \tag{2.16}$$

where σ_p is the Stefan–Boltzman constant (1.5204×10^{13} ph/s-cm^2-K^3). The π term in Eq. (2.16) converts the units to radiance (ph/s-cm^2-sr).